Module A4

COMPARING NUMBERS

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Introduction

Consider the fictional property Gunnadoo, a farm grazing cattle and sheep. In 1991 stock numbers indicated that times were good and food plentiful. By 1994 drought had taken its toll and Gunnadoo had reduced its herd numbers. As things pick up after the drought Gunnadoo is again building up stock numbers. Throughout this module we will look closely at the stocking patterns for this farm, comparing these numbers in different ways.

	1991	**1994**	**1997**
Cattle	500	100	300
Sheep	2 000	500	900

Of course, you may not live on a cattle or sheep station but the ability to accurately compare numbers or quantities is sure to come up sometime in your life. For example, have you ever seen these expressions.

On a bottle of mix-up cordial: Mix with water in the ratio 1:4

In the newspaper: Interest rates have fallen by a further 0.75%

On a map: Scale 1:1 000 000

At the supermarket: 6 for $1 or 20 cents each.

In a recipe book: Recipe for 12 biscuits and you wish to make 20.

Even though you may not recognise the above as comparisons at this stage, they are all types of mathematical comparisons.

On successful completion of this module you should be able to:

- make comparisons using subtraction;
- make comparisons using percentages;
- convert between percentages, fractions and decimals;
- solve problems using percentages including those involving interest; and
- make comparisons using division, with division expressed as ratios.

4.1 Comparing quantities by subtraction

Recall:
Module 2
Section 2.2.1

We have already looked at comparing two numbers in module 2.

Do you remember we talked about one number being smaller or greater than another? This is one type of comparison that we will now explore further.

It is sometimes enough to know that one quantity is greater or smaller than another, but often we need to know by how much these quantities differ.

Example

Consider the 1991 stock numbers for Gunnadoo.

Cattle	Sheep
500	2 000

You can probably see that the number of sheep is **1 500 more than** the number of cattle. To obtain this we have **subtracted.**

That is, 2 000 − 500 = 1 500

Note that we have put the larger of the numbers first. We could also say that the number of cattle is **1 500 less than** the number of sheep.

4.2 Comparing quantities by division

There are a number of methods for comparing numbers which we could classify as comparison by **division**. Some of these you might already be familiar with, some may be new to you. Let's look at the general case firstly.

Consider our cattle and sheep numbers for Gunnadoo in 1991 again.

If we were to divide the number of sheep (the larger number) by the number of cattle we get:

$$\frac{\text{Number of sheep}}{\text{Number of cattle}} = \frac{2\ 000}{500} = 4$$

We can say that there are **4 times as many** sheep as there are cattle.

If we were to divide the number of cattle (the smaller number) by the number of sheep we get:

$$\frac{\text{Number of cattle}}{\text{Number of sheep}} = \frac{500}{2\ 000} = \frac{1}{4}$$

We can say that there are $\frac{1}{4}$ **as many** cattle as there are sheep.

Example

Ms Welltodo earns \$1 380 a week while Mr Barelycando earns \$460 per week. Compare the earnings of these two workers by subtraction and by division.

Using the subtraction method: \$1 380 – \$460 = \$920

We can say that Ms Welltodo earns \$920 more per week than Mr Barelycando.

Using the division method: $\$1\ 380 \div \$460 = \dfrac{\$1\ 380}{\$460} = 3$

We can also say that Ms Welltodo earns 3 times as much per week as Mr Barelycando.

Looking at this the other way, we could also say that Mr Barelycando earns $\frac{1}{3}$ of the amount that Ms Welltodo earns.

Have you noticed the different words we have used here to distinguish between using division and subtraction. When we use division our answer represents 'times as many' or 'times as much' but when using subtraction our answer represents 'more than' or 'less than'.

Look at these figures, from the *Australian Council on Smoking and Health.*

Fags and figures

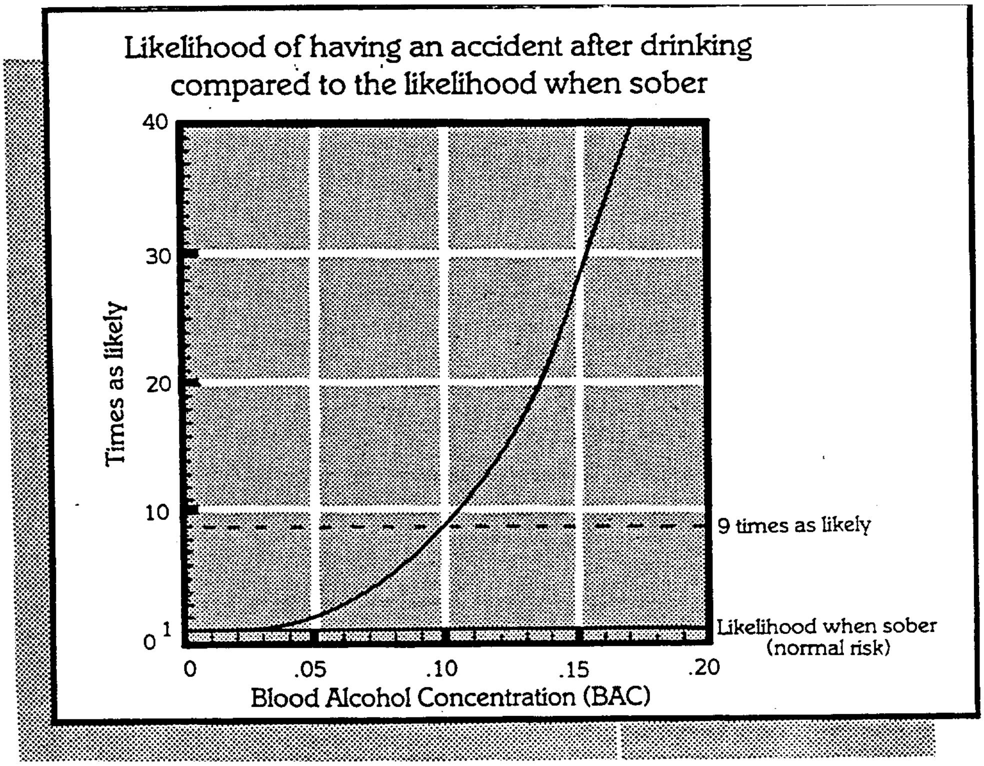
In people aged between 40 and 49 who smoke 20 - 30 cigarettes a day, the risk of heart attack is increased -- **4 times**.

Women who smoke 25 cigarettes a day and use an oral contraceptive increase their risk of heart attack by up to -- **39 times**.

After a heart attack, the likelihood of a further attack in a patient who continues to smoke, compared with their age group among nonsmokers is -- **2 or 3 times.**

What the first statement is saying is that you are **4 times** as likely to die of a heart attack, if you are in this age bracket and smoke this number of cigarettes, than someone in this age bracket who doesn't smoke. You may have heard the expression a 'four fold increase' in the chance of dying.

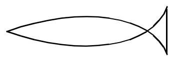
In another drug related example, we can look at the relationship between blood alcohol concentration (BAC) and the likelihood of having an accident. Can you see from the following graph that the likelihood of having an accident with a Blood Alcohol Concentration (BAC) of 0.05 (the legal limit in all states of Australia) is **2 times** the likelihood when sober. If the BAC is 0.10 you are **9 times** as likely to have an accident. Even a small amount of alcohol increases the likelihood of having an accident.

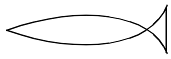


We haven't yet looked at interpreting graphs, so if you have trouble interpreting this graph, come back to it after you have completed module 5.

Activity 4.1

1. Consider the following maximum lengths for the given fish

   Bream  (50 cm)

   Blue Groper  (150 cm)

   Compare the lengths of the two fish by subtraction and by division. Express your answers as sentences.

2. Consider the following table that you have seen in a previous module. If you need to check on your understanding of scientific notation you should do so now.

Colour of hair	**Number of hairs**
Black or brown	1.05×10^5
Blond	1.4×10^5
Red	9×10^4

   Compare the number of hairs on the heads of blondes and red heads by subtraction and by division.

3. Suppose that 2 cans of white paint are mixed with 3 cans of red paint to make a new shade of pink. Compare, by subtraction and by division, the amounts of the different coloured paints used to make the pink paint.

4. According to the ***Guinness Book of Records***, the tallest person for whom there is irrefutable evidence is Robert Wadlow. When last measured in 1940, just before his death, Wadlow had reached a height of 2.72 metres. The shortest mature human for whom evidence exists, is Gul Mohammed from India. When examined in 1990 at the age of 33, he was 57 centimetres in height. Compare, by subtraction and by division, the heights of Robert Wadlow and Gul Mohammed.

5. Have you ever wondered why you see lightning before you hear it? Light travels at approximately 1.1×10^9 kilometres per hour while sound travels at roughly 1.2×10^3 kilometres per hour. How many times faster than sound is the light travelling?

4.2.1 Percentages

Percentages were first used in the fifteenth century for calculating interest, profits and losses. Currently they have a much broader application as indicated in the newspaper items below.

... maths scores of the children who had learnt to play the piano leapt by 34 %

Surfboards, bodyboards and surfskis caused 61% of injuries....

6% p a
Term Deposit

When you insure with NRMA you receive a 10% discount on each policy

Clearance
10 to 50% off everything.

Converting from a fraction to a percentage

Let's consider a person with the following marks over their first three assignments for a particular subject.

Assignment 1	Assignment 2	Assignment 3
$\frac{17}{20}$	$\frac{8}{10}$	$\frac{21}{25}$

Because the assignments are all out of different marks, it is very hard just looking at these figures to decide which assignment has given the student the best result. A very convenient method of comparing these results is to make them all out of the same mark....100.

Recall:
Module 2
Section 2.5.1

Assignment 1 $\frac{17}{20} = \frac{17 \times 5}{20 \times 5} = \frac{85}{100}$

Assignment 2 $\frac{8}{10} = \frac{8 \times 10}{10 \times 10} = \frac{80}{100}$

Assignment 3 $\frac{21}{25} = \frac{21 \times 4}{25 \times 4} = \frac{84}{100}$

It is now quite easy to compare results and see that the first assignment gave the student the best result. We call the resulting comparison with 100 a **percentage** (from *per cent* meaning out of one hundred). We represent a percentage by the symbol %.

The percentage sign is thought to have been derived as an economy measure when recording in the old counting houses; writing in the fraction $\frac{25}{100}$ of a cargo would take two lines of parchment, and hence the 100 denominator was put alongside the 25 and rearranged to become %.

Back to the above student's assignment marks expressed as percentages:

Assignment 1	85%	Say 85 percent, meaning 85 out of 100.
Assignment 2	80%	
Assignment 3	84%	

Rather than writing the assignment mark as a mark out of 100 we can simply multiply the fraction by 100 to get the value of the percentage.

Assignment 1 $\frac{17}{20} \times 100\% = \frac{17 \times \overset{5}{\cancel{100}}\%}{\underset{1}{\cancel{20}} \times 1} = 85\%$

Assignment 2 $\frac{8}{10} \times 100\% = \frac{8 \times 10\cancel{0}\%}{1\cancel{0} \times 1} = 80\%$

Assignment 3 $\frac{21}{25} \times 100\% = \frac{21 \times \overset{4}{\cancel{100}}\%}{\underset{1}{\cancel{25}} \times 1} = 84\%$

Note that we are not changing the value of the fraction, just the look of it:

$\frac{17}{20} = \frac{85}{100} = 85\%$ the percent sign meaning out of 100.

We can generalise this process.

When converting to a percentage, form a fraction and multiply by 100%.

Example

Let's return to Gunnadoo. In which year did the farm have the greatest percentage of cattle compared to the entire stock?

We must firstly form a fraction and then multiply by 100% for each year.

Percentage of cattle in 1991 $= \frac{\text{Number of Cattle} \times 100\%}{\text{Total Number of Stock}}$

$= \frac{\overset{20}{\cancel{500}} \times 1\cancel{00}\%}{\underset{1}{\cancel{2\,500}}}$

$= 20\%$

$$\text{Percentage of cattle in 1994} = \frac{\text{Number of Cattle} \times 100\%}{\text{Total Number of Stock}}$$

$$= \frac{\overset{50}{\cancel{100}} \times 1\cancel{00}\%}{\underset{3}{\cancel{600}}}$$

$$\approx 17\%$$

$$\text{Percentage of cattle in 1997} = \frac{\text{Number of Cattle} \times 100\%}{\text{Total Number of Stock}}$$

$$= \frac{\overset{25}{\cancel{300}} \times 1\cancel{00}\%}{\underset{1}{\cancel{1\,200}}}$$

$$= 25\%$$

We can see from these figures that in 1997 Gunnadoo had the highest percentage of cattle.

Example

You pour out 200 mL from a bottle containing 1 000 mL. What percentage of the liquid did you pour out?

$$\text{Percentage poured} = \frac{\text{amount poured out}}{\text{total amount}} \times 100\%$$

$$= \frac{20\cancel{0}\ \cancel{\text{mL}} \times 1\cancel{00}\%}{1\cancel{000}\ \cancel{\text{mL}}}$$

$$= 20\%$$

To do this on your calculator you would press the following keys.

2 0 0 ÷ 1 0 0 0 × 1 0 0 =

The display should of course read 20 and you will know to add the % sign.

As you can see it is often easier and quicker to do the calculation by hand using the technique of cancelling zeros.

Example

A team won 7 matches out of 8. What percentage did they win?

$$\text{Percentage won} = \frac{7}{8} \times 100\% = 87.5\%$$

(Check this on your calculator).

Example

Suppose I had painted 75 cm of a 3 m post. What percentage of the post has been painted?

As with previous examples, we cannot compare these two numbers while they are in differing units.

$$\text{Percentage painted} = \frac{75\text{ cm} \times 100\%}{3\text{ m}}$$

$$= \frac{\overset{25}{\cancel{75}}\text{ cm} \times 1\cancel{00}\%}{{}_{1}\cancel{3}\cancel{00}\text{ cm}}$$

$$= 25\%$$

(Check this on your calculator.)

Activity 4.2

1. Write as a percentage.
   (a) 8 out of 10
   (b) 250 mL out of 400 mL
   (c) 800 g out of 2 000 g
   (d) 25 cm out of 80 cm
   (e) \$25 out of \$60
   (f) 50 mL out of 2 L
   (g) 2×10^4 light years out of 3.5×10^3 light years
2. In a class of 50 students taking a maths test, 45 passed. What percentage of the class passed?
3. In her life a green turtle lays an average of 1 800 eggs. Of these, some 1 395 don't hatch, 374 hatchlings quickly die, and of the remaining 31, only 3 live long enough to breed. What percentage of the green turtle's eggs hatch and live long enough to breed?
4. A survey of 200 people asking what cereal they ate for breakfast found the following results.Complete the table by calculating the percentage of people eating each type of cereal.

Cereal	Number of people	Percentage of people
Corn Flakes	50	
Rice Bubbles	42	
Nutri Grain	39	

Rolled Oats	23	
Muesli	11	
Coco Pops	10	
Other Cereals	25	

5. Consider the following figures for pedestrians killed in Queensland in 1996.

ALL AGES OVER 70 ALCOHOL LINKED

Killed - 55Killed - 14Killed - 16

Taken to hospital - 405Taken to hospital - 46Taken to hospital - 65

Treated at the scene - 381Treated at the scene - 20Treated at the scene - 43

Minor Injuries - 153Minor Injuries - 12Minor Injuries - 11

Total - 994Total - 92Total - 135

HOW PEDESTRIANS ARE KILLED

Crossing carriageway at traffic lights - 6

Crossing carriageway at pedestrian crossing - 1

Crossing carriageway with no pedestrian control - 30

Stationary on road side - 6

Walking against the traffic - 2

Walking with the traffic - 7

Playing on the roads - 1

Other - 2

(a) What percentage of all people involved in pedestrian accidents are killed?

(b) What percentage of the people involved in alcohol related accidents are taken to hospital?

(c) As a pedestrian, what is the most common way to be killed? What percentage of the total deaths, die in this way?

(d) We learn as a child that we should walk on the right hand side of the road so we are facing the approaching traffic. Do these figures still support this view? What figures did you compare to come to this decision?

What happens if we have a decimal that we wish to convert to a percentage? Since we can convert any fraction to a decimal equivalent, converting a decimal to a percentage is exactly the same process as converting a fraction to a percentage.

Example

Suppose that $\frac{3}{4}$ of the children under 6 believe in Santa Claus

$$\frac{3}{4} \times 100\% = 75\%$$

We are saying that 75% of the children under 6 believe in Santa Claus.

Now we know that $\frac{3}{4} = 3 \div 4 = 0.75$

and that $0.75 \times 100\% = 75\%$ Recall that multiplying by 100 moves the decimal point 2 places to the right

So, $\frac{3}{4} = 0.75 = 75\%$

So we can say that $\frac{3}{4}$ or 0.75 or 75% of the children under 6 believe in Santa Claus.

Example

Convert 0.93 to a percentage.

$$\begin{aligned} 0.93 &= 0.93 \times 100\% \\ &= 93\% \end{aligned}$$

Example

Convert 4.23 to a percentage.

$$\begin{aligned} 4.23 &= 4.23 \times 100\% \\ &= 423\% \end{aligned}$$

Converting from a percentage

Example

A well known department store is offering 15% off everything in the shop on one particular day. You are very interested in a new clock which is normally selling for \$35. What will the clock cost after the 15% **discount**?

Recall from a previous section that 15% means 15 per 100. As a fraction we can represent this as $\frac{15}{100}$

We could simplify this to be $\frac{3}{20}$

We could also represent $\frac{15}{100}$ as a decimal.

$$15\% = \frac{15}{100} = 0.15$$

You can work with a fraction or a decimal whichever you find most convenient.

Let's return to our question. We must firstly calculate how much we would save.

Discount offered is 15% of $35

We can write this as $15\% \times \$35$ We write the 'of' as a multiplication sign.

$$= \frac{15}{100} \times \$35$$
$$= \frac{3}{20} \times \$35$$
$$= \$5.25$$

We could use decimals in the same way.

$$15\% \times \$35$$
$$= \frac{15}{100} \times \$35$$
$$= 0.15 \times \$35$$
$$= \$5.25$$

Either way we have a discount of $5.25.

The price you will pay for the clock on 'discount day' is $35 – $5.25 = $29.75

Example

Convert 75% to a fraction and a decimal.

$$75\% = \frac{75}{100} = \frac{3}{4}$$
$$= 0.75$$

Example

Convert 345% to a fraction and a decimal.

$$345\% = \frac{345}{100} = \frac{69}{20}$$
$$= 3.45$$

Did you notice that we ended up with a number greater than 1. This is because the percentage was greater than 100%. We will come back to this in a later section.

Converting from a percentage to a fraction or decimal is a very useful skill. There are some percentages that we use so often that it is helpful to remember their fractional equivalents. This will also help in estimating answers which you should always be doing, even if only in your head.

Some of the common percentages are:

$$10\% = \frac{1}{10}$$

$$25\% = \frac{1}{4}$$

$$50\% = \frac{1}{2}$$

$$75\% = \frac{3}{4}$$

Example

Convert $25\frac{1}{2}$ % to a fraction and a decimal

You know that 25% = $\frac{1}{4}$ = 0.25 so you would expect this answer to be very close to this result.

Depending on the circumstances, you might do the question like this:

$$\begin{aligned} 25\tfrac{1}{2}\% &= \frac{25.5}{100} \\ &= 0.255 \end{aligned}$$

Recall that dividing by 100 moves the decimal point 2 places to the left.

This method is convenient if you are going on to do a calculation with your calculator.

Or, if calculating by hand you might do it like this:

$$\begin{aligned} 25\tfrac{1}{2}\% &= \frac{25.5}{100} \\ &= \frac{25.5 \times 10}{100 \times 10} \\ &= \frac{255}{1\ 000} \\ &= \frac{51}{200} \end{aligned}$$

To remove the decimal point, multiply top and bottom by 10.

Both of these answers are close to our estimate of $\frac{1}{4}$ or 0.25.

Let's now look at some more practical uses for percentages.

Example

Interest is paid by a bank at a rate of 4% p.a. (Note: p.a. means per annum which is really saying per year). If you invested \$2 000 for 2 years, how much money would you have?

You could think here that 10% would give \$200 a year and 5% is half of this at \$100, so 4% must be a bit less than \$100.

$$\begin{aligned}\text{Interest received for one year} &= 4\% \text{ of } \$2000 \\ &= \frac{4}{100} \times \$2000 \\ &= 0.04 \times \$2000 \\ &= \$80\end{aligned}$$

Interest for two years $= \$80 \times 2 = \$160$

At the end of the two years you would have \$2 000 + \$160 = \$2 160

Example

A hospital keeps on hand a 5% glucose solution (that is, a solution that is only 5% glucose). If the glucose container holds 500 mL what portion of the container is glucose?

5% of the container is actually glucose.

That is, 5% of 500 mL

$$\begin{aligned}&= \frac{5}{100} \times 500 \text{ mL} \\ &= \frac{5 \times 5\not{0}\not{0}}{1\not{0}\not{0}} \text{ mL} \\ &= 25 \text{ mL}\end{aligned}$$

So in a 500 mL container of 5% glucose solution, 25 mL would be glucose.

Example

In a recent survey of adult students returning to studying mathematics it was found that 49% of these students expressed some anxiety about this return to mathematics study. If the group consisted of 63 students how many were expressing some anxiety?

Let's think about our answer before continuing. 49% is about 50% which we know is $\frac{1}{2}$.

If half the students are expressing anxiety this is about 31 or 32 students.

Now to the actual calculation.

We are to find 49% of 63

$$
\begin{aligned}
& 49\% \times 63 \\
= \ & 0.49 \times 63 \\
= \ & 30.87 \qquad \text{We can't have a part of a student so we will round up.} \\
\approx \ & 31
\end{aligned}
$$

So approximately 31 of the 63 students expressed some degree of anxiety.

Example

Let's look again at Gunnadoo. When the drought began to have an effect and stock numbers were reduced, what percentage of the cattle and sheep were removed?

Whenever we are looking at a **percentage reduction** or a **percentage increase** we must put the amount of the increase or decrease over the **original amount**.

Let's look at cattle firstly on Gunnadoo.

Number of cattle removed = 400 (500 – 100) This is the amount of the reduction

Original number of cattle = 500

If 400 out of 500 of the cattle were removed the percentage reduction is going to be quite high.

$$
\begin{aligned}
\text{The percentage decrease in cattle numbers} &= \frac{\text{Amount of decrease}}{\text{Original number}} \\
&= \frac{400}{500} \times 100\% \\
&= 80\%
\end{aligned}
$$

$$
\begin{aligned}
\text{The percentage decrease in sheep numbers} &= \frac{\text{Amount of decrease}}{\text{Original number}} \\
&= \frac{\overset{75}{\cancel{1\,500}}}{\underset{1}{\cancel{2\,000}}} \times 1\cancel{00}\% \\
&= 75\%
\end{aligned}
$$

On Gunnadoo the cattle were reduced by 80% while the sheep were reduced by 75%. The cattle have been reduced to a greater extent than the sheep.

Activity 4.3

1. Complete the following table.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{2}{5}$		
	0.25	
		80%
$\frac{3}{8}$		
	0.78	
$\frac{1}{3}$		$33\frac{1}{3}\%$

2. Calculate
   (a) 12% of 250 mL
   (b) 90% of $75
   (c) 30% of 645 g
   (d) 250% of $16.40
   (e) 2% of 900 mL
   (f) 15.6% of 300 mL
   (g) 5.5% of 350 g
   (h) 70.5% of 400 mL

3. Safety experts say that 60% of children's traffic injuries could be prevented by the use of child-restraint seats. If 6 100 children are injured each year in traffic accidents, how many injuries could be prevented with the use of child-restraint seats?

4. The longest bone in the human body is the thigh bone or femur. It normally measures about 27.5% of a person's height. Calculate the approximate length of your femur.

5. When the rains came to Gunnadoo and things were looking better, restocking began. What percentage increase in cattle and then sheep numbers took place in 1997?

The following questions are a little more involved. They include explanations as well as calculations for you to complete.

6. On a bank housing loan the interest is calculated on a reducing balance. That is, you only pay interest on the amount of money you still have owing. One such housing loan has an outstanding balance of $62 347.65 on the 1st July. Loan payments of $865 are paid on the 20th of each month and interest is calculated on the last day of the month. The current yearly interest rate is 8.2%

Date	Particulars	Debit	Credit	Balance
1 July				62 347.65
20 July	Loan payment		865.00	61 482.65
31 July	Interest	432.08		61 914.73
20 August	Loan payment		865.00	61 049.73
31 August	Interest			
20 September	Loan payment		865.00	
30 September	Interest			

As a loan is basically an overdrawn (money owing) situation, a payment is shown as a credit and is subtracted from the balance (amount owing). Interest is shown as a debit and is added to the balance.

Interest is calculated on a daily balance and charged for the number of days in the charging period.

Let's look at how the interest charged on July 31st was calculated.

The balance on this account was $62 347.65 from July 1 to July 20 when the loan payment was made. This amount was owed for 20 days.

The balance then was $61 482.65 until July 31 when the interest for July was calculated. This amount was owed for 11 days.

To summarise:$62 347.65 owed for 20 days
$61 482.65 owed for 11 days

Check that all 31 days in July have been accounted for.

Now to calculate the interest the bank will charge. The interest is calculated by finding the daily rate for any given balance and then multiplying by the number of days.

Let's consider the first balance. The interest for the whole year on this balance would be:

$\$62\ 347.65 \times \frac{8.2}{100} = \$5\ 112.5073$ Never round off until the end.

Now work out the daily rate: $\$5\ 112.5073 \div 365 = \$14.00686932$

Calculate for the number of days owing.
$\$14.00686932 \times 20 = \$280.1373863$

Now do the same for the other amount.

Yearly: $\$61\,482.65 \times \frac{8.2}{100} = \$5\,041.5773$

Daily: $\$5\,041.5773 \div 365 = \$13.81254055$

For 11 days: $\$13.81254055 \times 11 = \$151.937946$

Now add the two amounts together to find the total interest charged.

$$\$280.1373863 + \$151.937946 = \$432.0753323....$$

Now we can round off these numbers. The amount of interest charged is $432.08

You can see from these calculations that by making payments more frequently you owe a little bit less money each time and the interest for the month will be a little bit less. Over the period of a housing loan these saving become quite significant.

(a)By calculating the interest charges for August and September, complete the above bank statement up to the end of September.

7. For those of you who are now studying, have been or will be studying the 'Self Development' modules of the Tertiary Preparation Program, there are some interesting percentage applications for you to look at. Let's look to module 4 firstly and the tables on projected growth in a variety of employment areas. Some of these tables are reproduced below.

Projected Employment Growth by Major Group, 1992-2001 (percent)

Occupation	**Total Change in occupational employment (%)**
Managers	40.0
Professionals	38.2
Para-professionals	35.5
Tradespersons	21.1
Clerks	21.9
Sales workers	48.5
Machine operators	–1.9
Labourers	8.8
Total	**27.4**

(Source: Projections prepared for the ***Workforce 2005 Report***)

(a) Which group of workers is projected to show the greatest increase in employment numbers?

(b) Why do you think the increase in employment numbers for the machine operators is a negative percentage? Write a sentence to explain the negative percentage.

(c) What is the difference between the highest and lowest increases in employment numbers?

There were 120 different occupations included in these eight groupings. The full details are in module 4 of the self development modules. Included here are two groups in detail.

Occupational employment projections 1993-94 to 2004-05

Occupation	Total change in employment (%)	1993-94 Number employed	2004-5 Number employed
Managers			
Legislators or Gov Officials	41.1	2 600	3 700
General managers	73.1	32 900	57 000
Specialist managers	54.1	197 500	304 300
Farmers & farm managers	27.4	247 000	314 700
Managing supervisors	38.2	404 200	558 400
Total Managers	**40.0**	**884 200**	**1 238 100**
Machine Operators			
Road and rail drivers	2.6	259 300	266 000
Mobile plant operators	3.4	102 000	105 500
Stationary plant operators	–6.9	52 700	49 000
Machine operators	–12.4	138 300	121 200
Total Machine Operators	**–1.9**	**552 400**	**541 600**

(Source: Deet 1995, pp. 139-42)

(d) Let's consider the managers first. The total increase in numbers is expected to be 40%. Can you show how this figure was calculated? Remember, it is a percentage increase in the number of people employed. Round your answer to one decimal place.

(e) For the second employment group, look at the figures for the *stationary plant operators*. As for an earlier question there has been a decrease in the number of people employed in this area. Calculate the percentage increase in numbers of stationary plant operators. Round your answer to one decimal place.

Did you get the same answer as in the table? You need to be very careful when gathering information from any data that is presented to you.

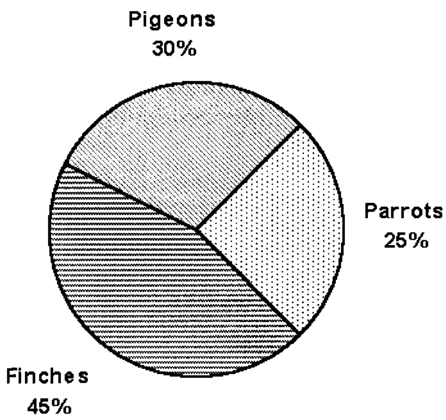
(f) Calculate the percentage increase in the *total number of machine operators*. Round your answer to one decimal place.

Did you get the same answer as in the table this time? If you did, have a close look at your working again.

What error has the person that developed this table made when calculating this answer?

8. A bird fancier with 300 birds was asked to estimate the percentage of the various types of birds in his aviary. The information is given below.

Types of Birds kept by a Bird Fancier



The **pie chart** above is a very convenient way to represent information. It shows, in this case, the percentage of each type of bird in the aviary.

The entire pie chart represents the **total** number of birds in the aviary.

(a) Calculate the number of each type of bird in the aviary.

Percentages in pie charts

Let's look in a little more detail at the pie chart from the last activity.

We said in the question that the pie chart represented the total number of birds in the aviary. Add up the percentages for each type of bird.

What total percentage did you get?...

You should have arrived at 100% which represented the total number of birds. In fact we use 100% to represent the whole of whatever we are referring to.

If we were referring to your weekly income, then 100% of your income would be everything that you were paid.

If we were talking about 100% of the stock on Gunnadoo, then we would be referring to all the cattle and sheep on the farm.

Let's look at how we could have drawn the above pie chart.

Suppose this is the information from the bird fancier.

Type of bird	Number
Parrots	75
Finches	135
Pigeons	90

To produce a pie chart for this information, the first step is to find the proportion (fraction) of each type of bird.

To do this we must firstly know the total number of birds we are dealing with. From the original question or by adding up the number column of the table we know that there were 300 birds altogether.

Proportion of Parrots $= \frac{75}{300} = \frac{1}{4} = 25\%$

Proportion of Finches $= \frac{135}{300} = \frac{9}{20} = 45\%$

Proportion of Pigeons $= \frac{90}{300} = \frac{3}{10} = 30\%$

You should **check** that your percentages all add to 100% to check that you haven't made any errors in your calculations.

We will now have to work out the size of the different **sectors** (pie shaped pieces) of the circle.

The full circle will represent the total number of birds in the aviary. You will also need to know that there are **360° in a circle**.

As parrots make up 25% of the aviary population then parrots should make up 25% of the circle. That is, they make up 25% of the 360° in the circle.

Angle for parrots: 25% of 360° = 0.25 × 360° = 90°

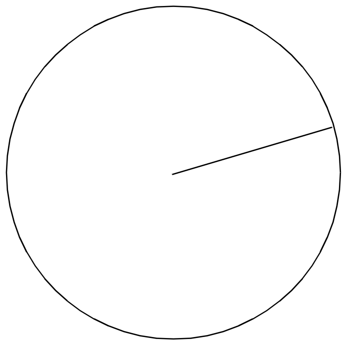
See if you can complete the calculations for the other birds.

Angle for finches: 45% of 360° = ☐ × ☐ = ☐

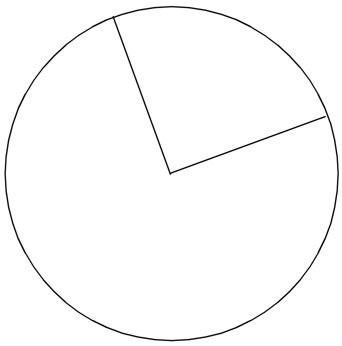
Angle for pigeons: 30% of ☐ = ☐ × ☐ = ☐

Did you get 162° and 108° for finches and pigeons respectively?

Now to the actual drawing of the pie chart. Using a pair of compasses  draw a circle. Clearly mark the centre. Draw a line from the centre to the edge (we call this line a **radius**).



To mark in the sector for parrots use a protractor to make an angle of 90°

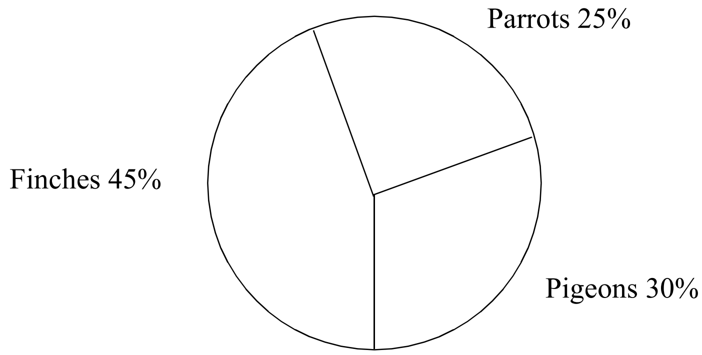


If you are not familiar with the use of a protractor please refer to the appendix at the end of this module.

Now construct the remaining sectors.

Give your pie chart a title and the finished product should look like:

Types of birds kept by a bird fancier



Activity 4.4

1. The following table shows the percentage of people preferring their steak to be cooked in each of the following ways.

Degree of 'doneness'	Overall (%)
Rare	5
Medium/rare	19
Medium	28
Medium/well done	18
Well done	30
	100

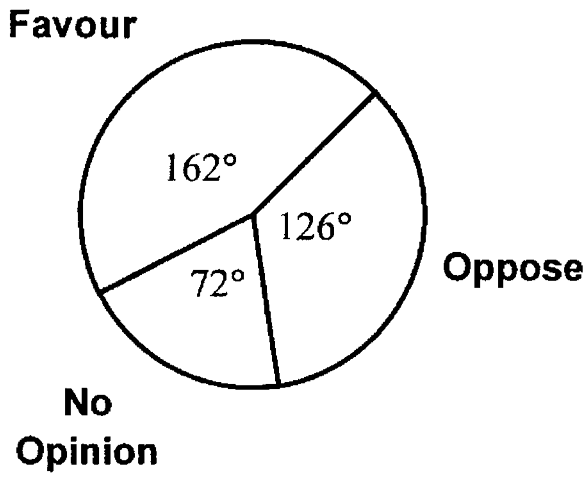
(a) Construct a pie chart using the above information. Round your angles to the nearest degree, checking to see that you only end up with 360°

(b) If 3 450 restaurant patrons were surveyed, how many of these would you expect to order their steak medium?

(c) Because everybody has a different idea of what is meant by each of the above categories, a recent study found that about 31% of people believed the steaks they received were not cooked to the ordered degree of 'doneness'. How many of the 3 450 restaurant patrons surveyed in part (b) would you expect to be unhappy with their steak?

2. Forty primary school teachers were surveyed about a proposal to begin and end the school day half an hour earlier. Their responses were collated and are partly shown on the pie chart below.

Teachers' attitude to change in hours



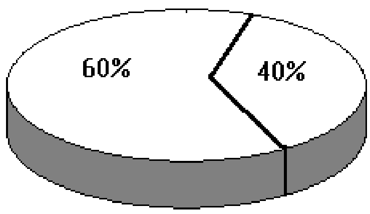
(a) Calculate the percentage of teachers in each of the three categories.

(b) How many of the surveyed teachers opposed the change in hours?

3. Following are two pie charts that have appeared in advertising material. Neither are accurate examples of the use of the pie chart. Write a few sentences commenting on the problems with each of the charts.

(a)

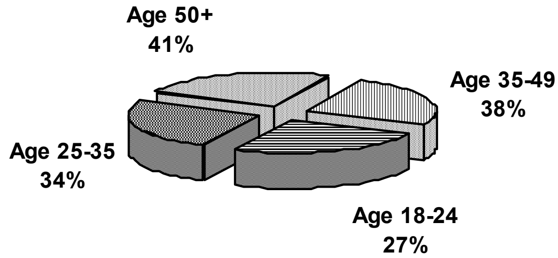
Heart Tablet $40.00



(Health Insurance Commission 06/93)

(b)

Heartburn, indigestion & age



(Source: *Discover Better Health*, A pamphlet from your local chemist)

4.2.2 Ratios

Another method of comparison by division is the idea of a ratio. A ratio is a comparison of two or more related amounts.

Consider the pink paint made in a previous activity by mixing 2 cans of white paint with 3 cans of red paint. This will make a total of 5 cans of pink paint. However, suppose that 10 cans of this pink paint were needed. Then the amounts of each colour (white and red) will have to be doubled. Provided that **both** colours are doubled, the same shade of pink will be produced. It is important to recognise that the colour of the mixed paint depends on the **ratio** of the white paint to the red paint. In this case, the ratio of white paint to red paint is

2 : 3 Read this as *'2 is to 3'*

2 of white is to 3 of red

A **ratio** is a statement of **related** amounts.

Ratio comes from the Latin *ar* meaning 'to fit together'. The Latin *ratio* developed from the idea of fitting numbers together in the sense of comparing them. Related to these words meaning fitting together are, arthritis - inflammation in the fitting together of the joints, adorn - make more appealing by adding decoration and read - the fitting together of words to give understanding.

A ratio does not contain any units. In fact the same shade of pink could be made by mixing 2 **cups** of white with 3 **cups** of red or by mixing 2 **litres** of white with 3 **litres** of red. As long as the two colours stay in the same relative amounts we will always get the same shade of pink.

As we saw above, 10 cans of this paint could be made by mixing 4 cans of white with 6 cans of red.

We could express this as the ratio 4 : 6

Just as we can convert fractions to their simplest form, we can do the same with ratios.

Consider the ratio of sheep to cattle numbers on Gunnadoo in 1991.

The ratio of sheep to cattle 2 000 : 500 Dividing both sides by 100 or cancelling zeros.

20 : 5

20 ÷ 5 : 5 ÷ 5 Dividing both sides by 5

4 : 1

Therefore 2 000 : 500 is the same ratio as 4 : 1. We are saying that for every cow there are 4 sheep. Or we could say, just as we did before, that there are 4 times as many sheep as there are cattle.

Example

Simplify the ratio 15 : 10

15 : 10

15 ÷ 5 : 10 ÷ 5

3 : 2 There is nothing further that will divide into both 3 and 2 so this is in its simplest form.

Hence 15 : 10 is the same as 3 : 2 We call them **equivalent** ratios.

We can write this as $15 : 10 \equiv 3 : 2$ The symbol $\equiv$ means 'is equivalent to'.

So far in this module we have only looked at comparing two quantities. A ratio allows us to compare any number of quantities.

Consider a particular mix of concrete that is made by mixing cement, sand and gravel in the ratio 1 : 3 : 4

1 : 3 : 4

1 part cement 3 parts sand 4 parts gravel

We can simplify a ratio of more than two quantities using the same methods as before.

Example

Simplify 20 : 16 : 12

20 :16 : 12

20 ÷ 2 : 16 ÷ 2 : 12 ÷ 2

10 : 8 : 6

10 ÷ 2 : 8 ÷ 2 : 6 ÷ 2

5 : 4 : 3

Don't worry if it takes you more than one step to reach the simplest form.

Example

Express 15 seconds to 2 minutes as a ratio in simplest form.

Before we can do this, each measurement must be expressed in the same unit. We could change both to seconds or both to minutes. We will choose seconds.

$$\begin{aligned}\text{2 minutes} &= 2 \times 60 \text{ seconds} \\ &= 120 \text{ seconds}\end{aligned}$$

The ratio then can be written:

15 : 120	Remember there are no units in a ratio.
$15 \div 15 : 120 \div 15$	Don't be concerned if you took more steps. It
1 : 8	doesn't matter how many steps you take.

So 15 seconds to 2 minutes can be written as the ratio 1 : 8

Example

Just as we can simplify ratios by dividing both sides by the same number, we can also multiply both sides by the same number to form an equivalent ratio.

Consider our pink paint from before. It was mixed white to red in the ratio 2 : 3

How many cans of red paint would be needed to make this shade of pink paint, if 6 cans of white paint are available?

$$\text{White : Red} \equiv 2 : 3 \equiv 6 : ?$$

What did we multiply 2 by to get to six? ....

You should have said 3 because $2 \times 3 = 6$. This was fairly easy to see. You could also have come up with this answer by dividing 2 into 6. That is $6 \div 2 = 3$ We will come back to this in a minute.

What you do to one part of the ratio you must do to the other, so

$$2 \times 3 : 3 \times 3 \equiv 6 : 9$$

If 6 cans of white paint were available, 9 cans of red paint would be needed to make the same shade of pink.

Using the same ratio of paints, how many red cans would I need if I had only 5 white cans?

That is $2 : 3 \equiv 5 : ?$

We ask ourselves the same question. What did I multiply 2 by to get 5? This is a little harder than the last example as this time it is not easy to see the answer. If we use the second method mentioned before, we will find the answer.

That is, $5 \div 2 = \frac{5}{2}$ So we must multiply both sides by $\frac{5}{2}$

$$2 \times \frac{5}{2} : 3 \times \frac{5}{2} \equiv 5 : \frac{15}{2} \equiv 5 : 7\frac{1}{2}$$

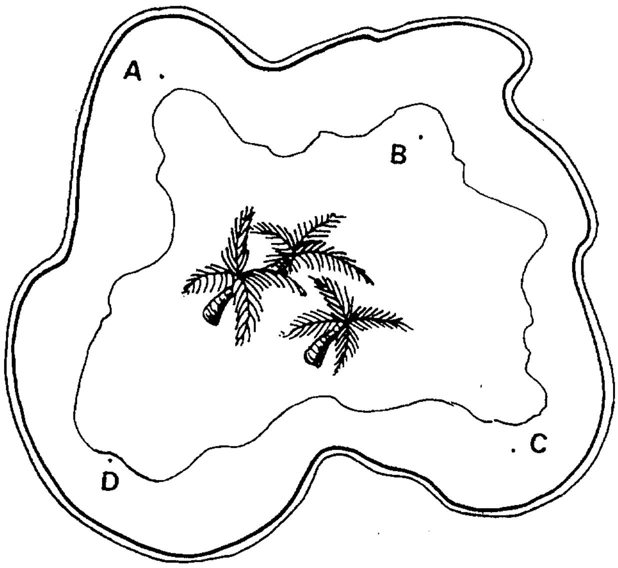
With 5 cans of white paint you would mix $7\frac{1}{2}$ cans of red paint.

Example

On a map or scale drawing you might see the scale written as 1 : 100 This ratio compares distance on the map or scale drawing to the actual distance. In this case, 1 unit on the map will be equivalent to 100 of the same unit in real life. For example, one centimetre of this map would represent 100 cm in real life. We could also state this as 1 cm is equivalent to 1 metre, since 100 cm is the same as 1 metre.

The following map shows the situation of four towns around an island. Find the distance between towns A and B.

Scale: 1 : 1 000 000



The scale on a map is 1 : 1 000 000, which we could think of as 1 cm on the map representing 1 000 000 cm of real distance. The distance between A and B measures 3.5 cm on the map.

$$1 : 1\,000\,000 \equiv 3.5 : ?$$

What did you multiply 1 by to get 3.5? You multiplied by 3.5.

$$1 \times 3.5 : 1\,000\,000 \times 3.5 \equiv 3.5 : 3\,500\,000$$

So the distance between A and B is 3.5 cm on the map giving a distance of 3 500 000 cm in real life. This is not an appropriate measure to express the distance between two towns. We should convert it to kilometres, a more appropriate measure.

Recall:
Module 3
Section 3.2.1

3 500 000 cm = 35 000 m = 35 000 × 10^{-3} km = 35 km

The distance between towns A and B is 35 kilometres.

Activity 4.5

1. Express the following as ratios in their simplest forms.

   (a) 21 : 15

   (b) 20 : 12

   (c) 350 g to 500 g

   (d) 90 mL to 300 mL

   (e) 5 mL to 3 L

   (f) 5 g to 10 kg

   (g) 2.2 kg to 500 g

   (h) 9.5 L to 50 mL

2. A human has 32 adult teeth. There are four different kinds of teeth. The teeth are classified as follows:

   incisors 4 pairs bicuspids 4 pairs

   canines 2 pairs molars 6 pairs

   (a) How many teeth are:

      (i) incisors

      (ii) canines

      (iii) bicuspids

      (iv) molars

   (b) Express the following as ratios in their simplest forms.

      (i) incisors to molars

      (ii) bicuspids to canines

      (iii) incisors to bicuspids

      (iv) molars to all teeth

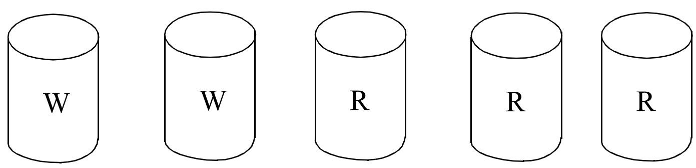
      (v) incisors to molars to all teeth.

3. If the basketball team wins 24 games out of 30 played
   (a) What fraction of the games are won?
   (b) How many games are lost?
   (c) What is the ratio of games lost to games won?
4. A company surveyed a group of people to find out information about diets. They found out that 35% of people skip breakfast. What is the ratio of people who eat breakfast to those who skip breakfast?
5. Find the unknown value that makes each of the following equivalent ratios true.
   (a) $8 : 5 \equiv 24 : ?$
   (b) $3 : 4 \equiv ? : 20$
   (c) $4 : 6 \equiv 10 : ?$
   (d) $7 : 3 \equiv ? : 4$
   (e) $? : 4 \equiv 2 : 1$
   (f) $\frac{4}{7} : \frac{3}{5} \equiv ? : \frac{6}{15}$
   (g) $2 : 0.75 \equiv ? : 0.25$
6. A garden compost is made up of loam, peat and sand in the ratio 7 : 4 : 2 If we use 4 buckets of sand, how much loam and peat are required?
7. From the map drawn in the previous example, find the distance between towns B and C.
8. I wish to draw a 1 : 500 scale drawing of a basketball court. I know that the exact dimensions of the court are 14 metres by 26 metres. What dimensions should I make the scale drawing?
9. The case load in a particular ward in the hospital is determined to be 12 patients to each two nurses. To have an equal case load, how many nurses are required for 70 patients?

Let's return to our pink paint.

If the pink paint is mixed in the ratio 2 : 3 (white to red) then 5 (that is, 2 + 3) cans of this pink paint is made up of 2 cans of white and 3 cans of red paint.

We can say that 2 out of 5 or $\frac{2}{5}$ of the cans will be white and 3 out of 5 or $\frac{3}{5}$ will be red.



Example

A herbicide mixture is $\frac{1}{5}$ chemical and the rest is water.

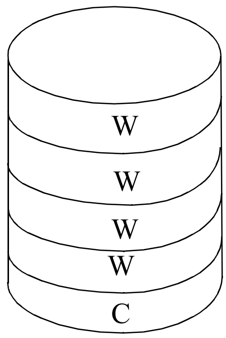
(a) What fraction of the herbicide is water?

(b) How much of each will have to be mixed to make 200 mL of herbicide?

(c) What is the ratio of chemical to water?

Solution:

(a) If the chemical makes up $\frac{1}{5}$ then the water must be $\frac{4}{5}$



$$\text{Fraction of water} = 1 - \frac{1}{5}$$
$$= \frac{5}{5} - \frac{1}{5}$$
$$= \frac{4}{5}$$

(b) To make 200 mL of herbicide:

chemical: $\frac{1}{5}$ of 200 mL

$$= \frac{1}{5} \times 200$$
$$= 40 \text{ mL}$$

water: $\frac{4}{5}$ of 200 mL

$$= \frac{4}{5} \times 200$$
$$= 160 \text{ mL}$$

Therefore we should mix 40 mL of chemical with 160 mL of water to make 200 mL of the herbicide. This looks reasonable as there should be a lot more water than chemical.

(c) The ratio of chemical to water is:

40 mL to 160 mL

40 : 160 We can simplify this further.

1 : 4

Remember that this means that 1 measure of chemical with 4 of the same measures of water will make herbicide of the same strength. Now, 1 part is chemical and 4 parts are water giving 5 parts altogether. We can also think of this as meaning that 1 part **in** 5 is chemical and 4 parts **in** 5 are water, as in the original question.

You may often see statistics written in this form.

Example

Some interesting facts from Larry Laudan's *Book of Risks* (New York, John Wiley & Sons, Inc, 1994) include:

- The risk that you will be injured this year is 1 in 3;
- The risk that you will injure yourself on a chair or bed this year is 1 in 400;
- The risk that you will injure yourself shaving this year is 1 in 7 000;
- The risk that you will die from falling out of bed this year is 1 in 2 000 000;
- The risk that you will be injured by your toilet is a mind boggling 1 in 6 500!!!

We can express each of the above risks as a ratio.

Risk of:

being injured:	1 : 2	That is, 1 in 3 (1 + 2)
being injured by chair or bed:	1 : 399	That is, 1 in 400 (1 + 399)
of a shaving injury:	1 : 6 999	That is, 1 in 7 000 (1 + 6 999)
death from falling from bed:	1 : 1 999 999	That is, 1 in 2 000 000 (1 + 1 999 999)
being injured by your toilet	1 : 6 499	That is, 1 in 6 500 (1 + 6 499)

Fancy your chances of winning Lotto? You might like to know that the chance of winning Lotto, with 45 numbers to choose from, is 1 in 8 145 060. If you bought six games each week, you could expect to wait about 26 000 years for your first win. Your chances of winning Lotto? **Fat Chance**!!!!!

Example

An inheritance of $8 235 is to be divided between Sally, Tony and Ursula in the ratio

2 : 3 : 4. How much will each person receive?

The ratio 2 : 3 : 4 consists of 9 parts (2 + 3 + 4) so we can say that:

Sally receives 2 parts out of the 9 or $\frac{2}{9}$

Tony receives 3 parts out of the 9 or $\frac{3}{9}$ and

Ursula receives 4 part out of the 9 or $\frac{4}{9}$

So Sally receives $\$8\ 235 \times \frac{2}{9} = \$1\ 830$

Tony receives $\$8\ 235 \times \frac{3}{9} = \$2\ 745$

Ursula receives $\$8\ 235 \times \frac{4}{9} = \$3\ 660$

To check that you have shared out all the money correctly, add your final figures.

$1 830 + $2 745 + $3 660 = $8 235

We have formed an equivalent ratio.

$2 : 3 : 4 \equiv 1\ 830 : 2\ 745 : 3\ 660$

Therefore from the inheritance, Sally receives $1 830, Tony receives $2 745 and Ursula receives $3 660.

Activity 4.6

When you are calculating answers for the following questions always check that your answer is reasonable.

1. An evening class contains 32 male students. If the ratio of males to females in the class is 4 : 3, how many females are in the class?
2. On Gunnadoo in 1970 the ratio of sheep to cattle was 13 : 5. If stock numbers totalled 1 980, how many cattle were on the property?
3. A fertilizer is made up of nitrogen, potash and phosphate in the ratio 2 : 3 : 4 respectively. How many kilograms of nitrogen would there be in a 27 kilogram bag of fertilizer?
4. Daniel and Tom help out at home by mowing the lawn. They are to be paid $10 for the job. Daniel is older and feels he did more work than Tom. Their mother decides that the money should be divided between Daniel and Tom in the ratio 3:2 How much money does each boy receive?
5. Human blood normally has a ratio of 1 000 : 1 between red blood cells and white blood cells. A sample of blood is found to have 2 125 white blood cells and 4 250 000 red blood cells. How does this compare with normal blood?
6. Following are the risks of dying this year from a job related accident or illness. Express each of these as a ratio.

   (a) An airline pilot: 1 in 1 100

   (b) In mining and agriculture: 1 in 2 300

   (c) In transportation: 1 in 4 500

   (d) In government: 1 in 11 000

   (e) In manufacturing 1 in 23 000

   (f) In sales 1 in 24 000

   (g) In office work 1 in 37 000

7. The mortality rate for a particular surgical procedure has been found to be 5 deaths for each 3 800 operations.

   (a) In one year, a total of 10 500 operations using this surgical procedure are conducted. How many deaths would be expected for this procedure over this time?

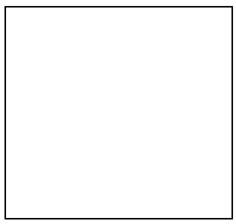
   (b) Research showed that 75 deaths occurred over a given period, for this type of surgery, how many operations would have taken place?

Ratios around us

Ratios are more than just a way of comparing facts and figures. Throughout history we have used ratios in the shapes of things around us to make things more pleasing to the eye or to be structurally sound. Let's now look at ratios associated with some common shapes.

Ratios in squares

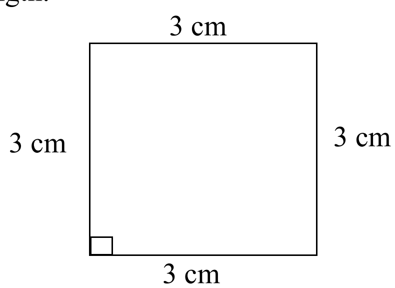
Firstly let's consider the square.



What do you already know about the square?

....

You might have said that all of its angles are right angles (90°) and all of its sides are the same length.



Notice the symbol we use to represent a right angle.

Recall from earlier work that the perimeter of any figure is the distance around the outside.

What is the perimeter of this square? ....

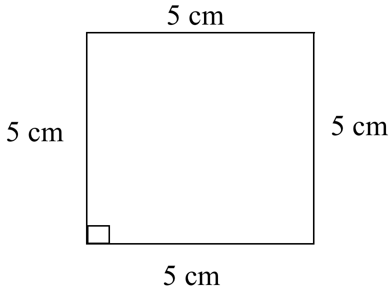
Write a ratio of perimeter to side length for this square. ....

Did you say that the perimeter was 3 cm + 3 cm + 3 cm + 3 cm = 12 cm

The ratio would then be 12 cm to 3 cm

That is 12 : 3 or when simplified this would become 4 : 1

Let's try that with another square.



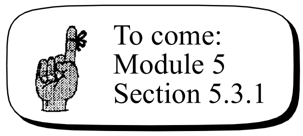
Find the ratio of the perimeter to the side length. ....

Did you get 5 + 5 + 5 + 5 : 5

$\equiv 20 : 5$

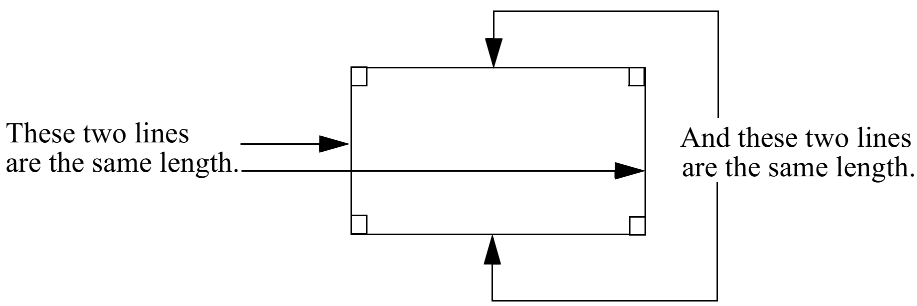
$\equiv 4 : 1$

In fact you should always get a ratio of 4 : 1 when you compare the perimeter of a square to its side length. That is to say that the perimeter is always 4 times the length of the side. We will look at a **formula** for this expression in a later module.

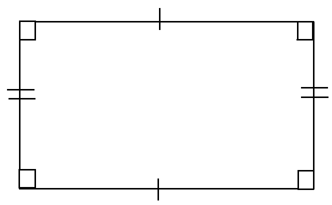


Ratios in rectangles

Let's firstly look at what it means to be a rectangle. Like a square, the rectangle has all angles equal to 90°, that is, they are all right angles. A rectangle also has opposite sides equal in length.



We represent lines with the same length by marking them with the same symbol.



Let's move on and look at a very special rectangle.

In studying art and architecture you may come across the golden ratio. The **golden ratio** is a number that dates back to at least 300 B.C. It was particularly important to the Greek civilizations where it appeared in many examples of art and architecture. The golden ratio is approximately equal to 1.62 : 1. A rectangle with its sides in this ratio is known as a **golden rectangle**. This rectangle was thought to be the most pleasing to the eye.

Ancient Greek architects in the 5th century B.C. were aware of the golden rectangles harmonious influence. The Parthenon at Athens is an example of the use of the golden rectangle in architecture. Many of Leonado da Vinci's paintings make use of the golden rectangle. In fact, he helped illustrate a book which dealt with the properties of the golden ratio.

Find the ratio of the length to the width of the following items. Express these ratios as 'something' : 1

(a) a normal envelope;

(b) a playing card;

(c) a postcard;

(d) a foolscap piece of paper; and

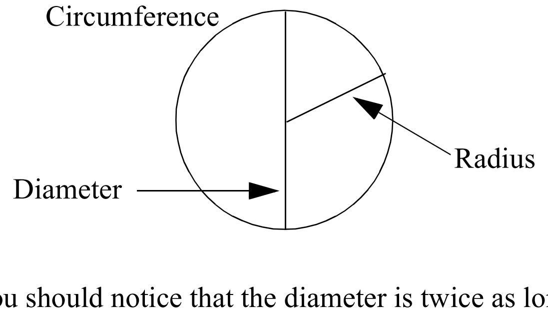
(e) find the ratio of your height to the height of your navel above the ground.

You should have found that most of these ratios fall close to the golden ratio. This ratio occurs in nature as well as the man made world. Look at things in your environment and see if you can find other instances of the use of the golden ratio.

Ratios in circles

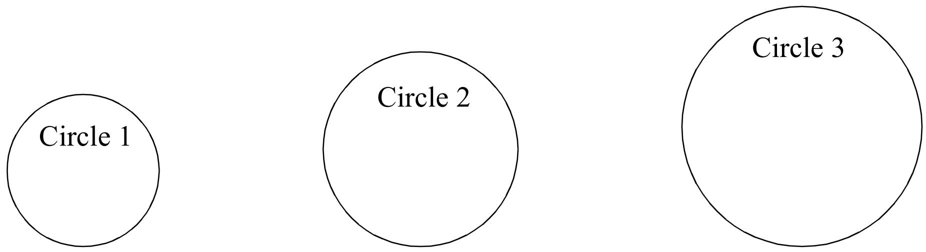
We have already come across circles in the section on drawing pie charts. Let's now look again at the circle and some of the names we give to the parts of the circle.

Remember that we have already talked about the **perimeter** of an object as the distance around the outside. This distance in a circle is given the special name **circumference**. The distance from the centre of the circle to the edge is called the **radius,** and finally the distance all the way across a circle through the centre is called the **diameter**.



You should notice that the diameter is twice as long as the radius.

By placing a piece of string around the edge, find the length of the circumference of the following circles. Then for each circle, find the length of the diameter. Measure these lengths as accurately as you can (to two decimal places if possible) and be sure you use the same unit for both measurements. Then for each circle write the ratio of the circumference to the diameter.



....

You will sometimes see a ratio written as a fraction. In this case the ratio has been expressed as a comparison to 1.

For example,

Consider the ratio 3 : 4

If we divide both sides of the ratio by 4 we get,

$$\frac{3}{4} : \frac{4}{4}$$

$$\frac{3}{4} : 1$$

$$0.75 : 1$$

You will sometimes see a ratio written just as a fraction or a decimal, for example $\frac{3}{4}$ or 0.75, but you would know that this meant it was compared to 1.

For each of the three ratios you found above, express them as a ratio to 1. Express your answers in decimal form.

Circle 1	Circle 2	Circle 3
....	....	....

You should have found that all your answers were about the same, depending on the accuracy of your calculations. In fact if your calculations were very accurate you should be able to recognise the number that you obtained for each circle.

The number in each case should be π at 3.14159..........

Recall:
Module 2
Section 2.6.5

You will recall from module 2 that π is an irrational number (as a decimal it is a never ending, non-repeating decimal).

So, circumference : diameter $\equiv \pi : 1$

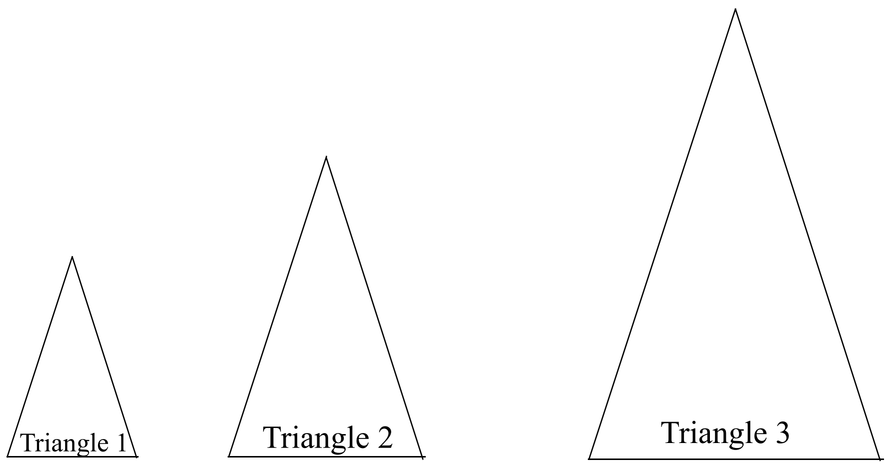
We say that the ratio of the circumference of a circle to its diameter is equal to π .Try this on some familiar circular items about your house.

We will use this information in the next module to develop a **formula** that will allow us to find the circumference of a circle without having to measure it.

The number pi (π) has fascinated people for thousands of years. It is thought that the Egyptians used a value for π when building the Great Pyramid of Cheops. It has been claimed that the height of the pyramid multiplied by π gives a result equal to the length of any two of the four sides measured at ground level.

Ratios in triangles

Just as we find the golden rectangle in art, architecture and nature, we also come across the **golden triangle** (we are referring to the geometric shape of a triangle, not the drug area of Asia). This is a triangle where two of the angles are 72° and the other angle is 36°. For the three golden triangles below, check with your protractor, that the two angles at the bottom of the triangle are 72° and that the angle at the top of the triangle is 36°.



What do all the angles in each of these triangle add up to? ....

Did you say 180°? That is right, there are **180° in every triangle**.

If you were to measure the lengths of the two upright sides you would find that they are the same length, while the base of the triangle is a different length. Check this now on the above triangles.

We call any triangle with two sides of equal length and two angles of equal size, an **isosceles triangle**.

Now, for the above three triangles, form a ratio of the upright side length to the base length.

Triangle 1	Triangle 2	Triangle 3
....	....	....

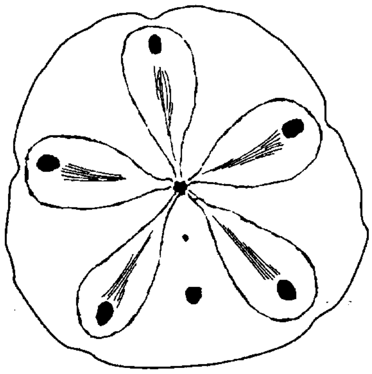
As we have done before, express these ratios as ‘something’ : 1

Triangle 1 Triangle 2 Triangle 3

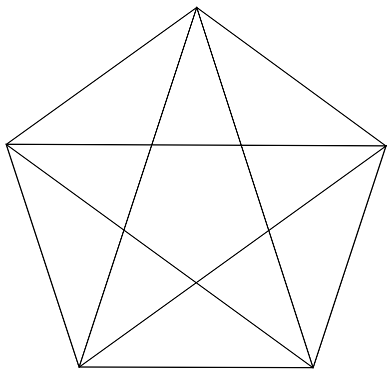
....

All your answers should be very close to the **golden ratio** which is approximately equal to 1.62 : 1

This figure shows a sand dollar, a sea creature often found washed up on beaches in the northern hemisphere.



It has five evenly spaced holes that, if connected in every possible way, form the figure shown below.

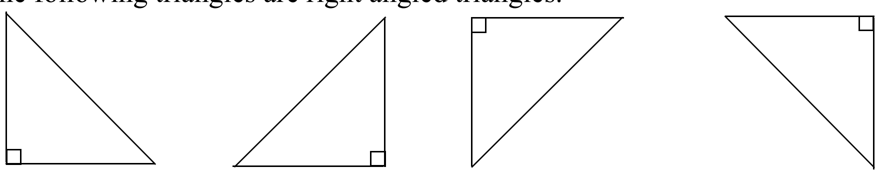


We call this five sided figure a **pentagon**. Using your protractor to measure angles, see how many golden triangles you can find in the above diagram. In fact you may find up to twenty different golden triangles.

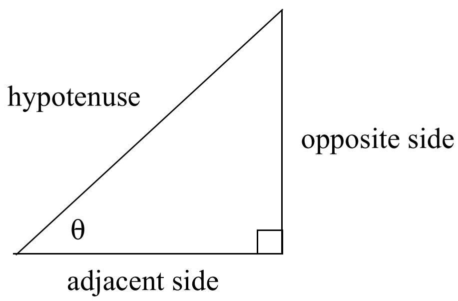
We have only briefly touched on the properties and occurrences of the golden ratio. If you would like to learn more about this interesting topic, you will find many books in the library worth reading.

Let's now look at another type of triangle. The triangle we will look at next is the **right angled triangle**. This simply means that one of the angles in the triangle is a right angle (90°).

All of the following triangles are right angled triangles.



Consider the following right angled triangle. Apart from the right angle, which is marked with a small square, another angle is usually marked. Often this angle is unknown or we want to represent the angle to be many values. In this case, we represent the angle with a symbol. Any symbol will do, but for this module we will use the Greek letter theta (θ).



We call the side opposite the right angle the **hypotenuse** (pronounced hi/pot/en/use). The side opposite the marked angle we call the **opposite side** and the remaining side is called the **adjacent side**. Note that the adjacent side will always be one of the sides that surround the marked angle.

Let's now look at some of the ratios of sides in the above triangle.

For the above triangle, measure the angle θ with your protractor ....

Measure the length of the side opposite the angle θ ....

Measure the length of the side adjacent to angle θ ....

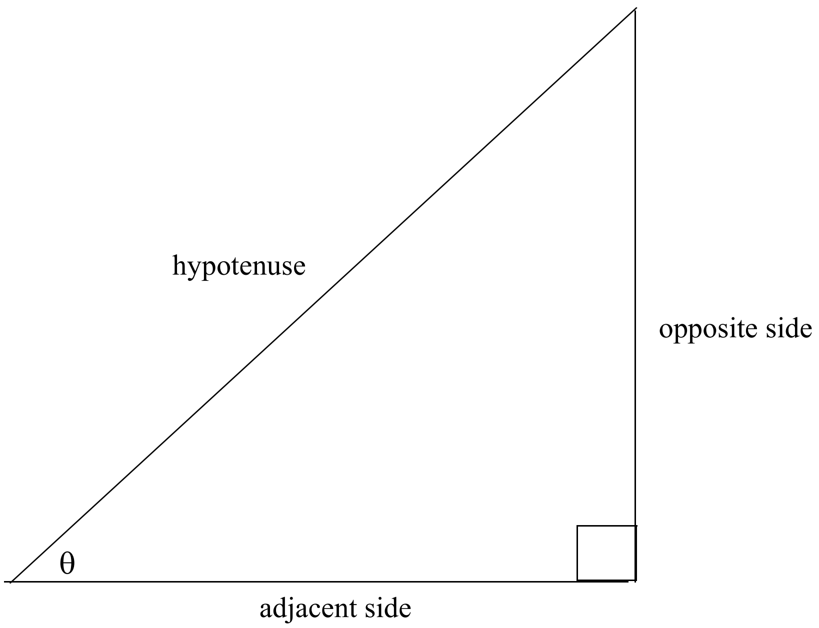
Measure the length of the hypotenuse. ....

It is usual practice when writing the ratios of side lengths in triangles to express the ratio as a fraction or decimal. Recall from earlier in this section that this means that we are expressing the ratio as a comparison with 1.

Find the ratio of the opposite side to the hypotenuse.

$$\text{Ratio} = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{\Box}{\Box} = \underline{\qquad\qquad}$$

Now repeat this procedure for the following triangle.



With your protractor, measure the angle θ. ....

Measure the length of the side opposite the angle θ ....

Measure the length of the side adjacent to angle θ ....

Measure the length of the hypotenuse. ....

$$\text{Ratio} = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{\square}{\square} = \_\_\_\_\_\_\_\_$$

What do you notice about the size of the angle and the ratio of the opposite side to the hypotenuse, compared to the last triangle?

....

You should have found that for this triangle, the angle was the same as in the previous triangle. You should also have found that the ratio was the same (or very close depending on the accuracy of your measurements and our diagrams).

In fact the ratio of the opposite side to the hypotenuse will always be the same provided that the angle θ is always the same.

We refer to the ratio of the **opposite side** to the **hypotenuse**, of a right angled triangle, as the **sine ratio** of the angle.

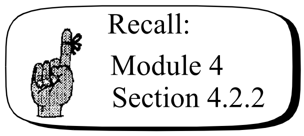
For an angle of 45°, the sine ratio will always be approximately equal to 0.707

What does this mean?

Let's write out this ratio so that it looks like our earlier ratios.

$$\text{opposite side} : \text{hypotenuse} \equiv 0.707 : 1$$

Remember back to when we looked at the ratio of sheep to cattle on Gunnadoo.



$$\text{sheep} : \text{cattle} \equiv 4 : 1$$

We are saying that for every cow there are 4 sheep. Or we could say that there are 4 times as many sheep as there are cattle.

So for our sine ratio, in a right angled triangle, with one angle equal to 45°:

$$\text{opposite side} : \text{hypotenuse} \equiv 0.707 : 1$$

We are saying that for every one unit of length (that might be a centimetre, a millimetre or any unit of length) on the hypotenuse, we have 0.707 units on the opposite side. Or we could say that the opposite side is 0.707 times the length of the hypotenuse. Can you now see the similarity of this ratio to the others we have looked at? The ratio still has the same meaning even though it is written only as a fraction or a decimal.

Example

Construct some right angled triangles with an angle of 30°.

Find the ratio of the opposite side to the hypotenuse, the sine ratio.

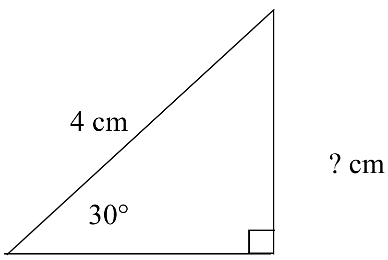
For an angle of 30°, the sine ratio will always equal 0.5. Does this agree with your findings?

We write this mathematically as

$\sin 30° = 0.5$ We write sin as an abbreviation for sine but still pronounce it as in 'sign'.

We are saying that in a triangle with an angle of 30°, that for each unit of length on the hypotenuse, we have half this unit of length on the opposite side.

Consider the following triangle (not drawn to scale).



In the above diagram, the hypotenuse is 4 cm long, how long is the opposite side?

We know that $\sin 30° = 0.5$ That is:

$$\text{opposite side} : \text{hypotenuse} \equiv 0.5 : 1$$

Now, we know the hypotenuse is 4 and we wish to find the length of the opposite side.

So, $? : 4 \equiv 0.5 : 1$

We now have a ratio statement very similar to many we have solved before.

$$? : 4 \equiv 0.5 : 1$$

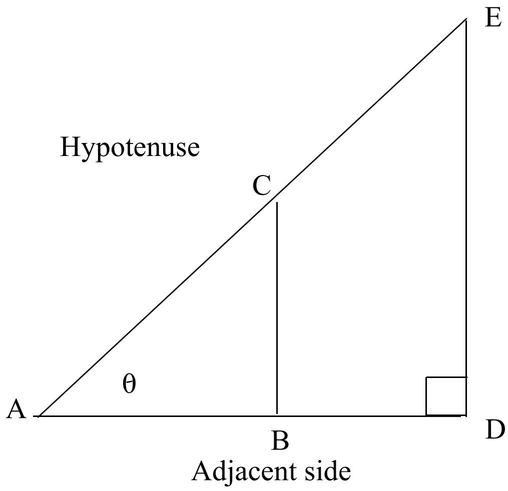
What did you multiply 1 by to get to 4? So, we must multiply 0.5 by 4 to find our unknown value.

$$0.5 : 1 \equiv 0.5 \times 4 : 1 \times 4 \equiv 2 : 4$$

The opposite side would have to be 2 cm long. We can check that, yes, opposite over hypotenuse or 2 over 4 does equal 0.5.

The same principles we have looked at for the sine ratio apply to the ratio of the length of the side **adjacent** to an angle and the **hypotenuse** of a right angled triangle. We call this the **cosine ratio** (pronounced co-sign and abbreviated to cos).

Try calculating the cosine ratio for the following triangle. We are using a method of naming triangles using letters. For example, we have below two triangles, triangle ABC and triangle ADE. If you move from one letter to the next in the name of the triangle, you will have a picture of which triangle is indicated. We also name sides in the same way. For example side DE means the side that runs from the letter D to the letter E. We could also have named this side ED. It doesn't matter which way you name the sides in this module.



For the triangle ABC Measure the angle θ = __________

Measure the adjacent side (AB) = __________

Measure the hypotenuse (AC) = __________

$$\text{The cosine ratio} = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$$

For this angle, the cosine ratio $= \dfrac{\square}{\square} =$

For the triangle ADE Measure the angle θ = __________

Measure the adjacent side (AD) = __________

Measure the hypotenuse (AE) = __________

For this angle, the cosine ratio = $\dfrac{\square}{\square}$ =

Hence, $\cos\ \theta$ = __________________

We would say this as cos theta or coz theta

If the cosine ratios were not similar for the two triangles, recheck your working.

There is one final ratio to consider.

The **tangent ratio** (tan for short) is the ratio of the length of the side **opposite** an angle to the length of the side **adjacent** to it, in a right angled triangle.

Find the tan ratios for each triangle in the last example.

For the triangle ABC Measure the angle θ = ___________

Measure the opposite side (CB) = __________

Measure the adjacent side (AB) = __________

The tangent ratio = $\dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

For this angle, the tangent ratio = _____________

For the triangle ADE Measure the angle θ = __________

Measure the opposite side (ED)= __________

Measure the adjacent side (AD)= __________

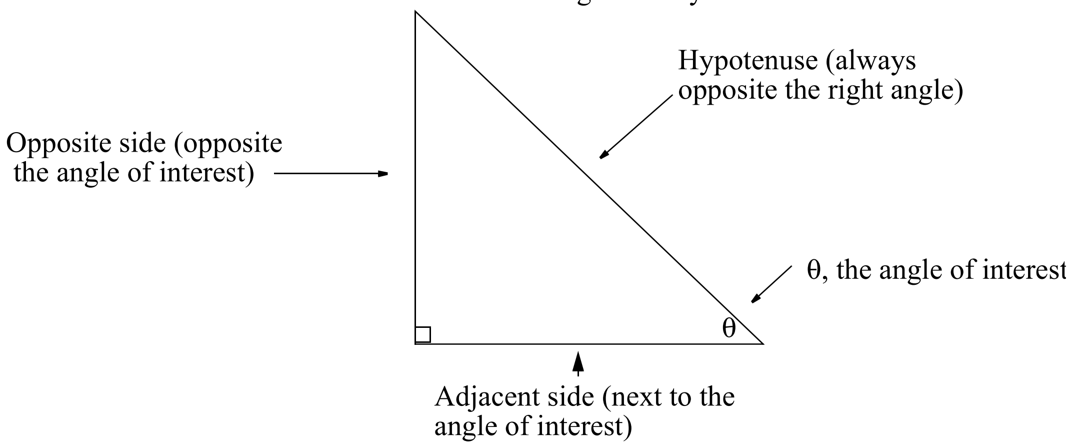
For this angle, the tangent ratio = ____________

Hence, $\tan\ \theta$ = _________________

We would say this as tan theta.

These ratios are three of the basic **trigonometric ratios**. We call this branch of mathematics **trigonometry** (trig for short). The word trigonometry names a science that was originally concerned with finding the lengths of the sides of triangles. Early astronomers used these ideas to solve problems related to the planets and stars. Today trigonometry is an essential part of navigation and surveying, but also has applications in business, drawing, design and building.

Let's summarise the work we have done so far on trigonometry.



For the angle θ,

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

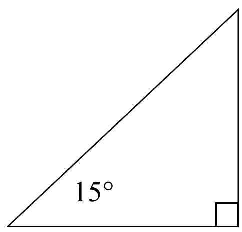
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

As you have seen, each angle has its own set of ratios. It will not be necessary to measure each time to find the ratio. Your calculator is able to find the sin, cos and tan of any angle that you require.

Example

Find the sin, cos and tan ratios for the angle marked in the following triangle.



Find the sin, cos and tan keys on your calculator

Before we move on to finding these values check that your calculator is in degrees. If it is in degrees it should have DEG written along the top of the display.

If it doesn't have DEG, press the MODE key and then the key for degrees.

To get the sine ratio for 15° press: sin 1 5 = on your calculator.

The display should read 0.258819045

To get cos 15° press 

on your calculator.

The display should read 0.965925826

And finally tan 15° = 0.267949192

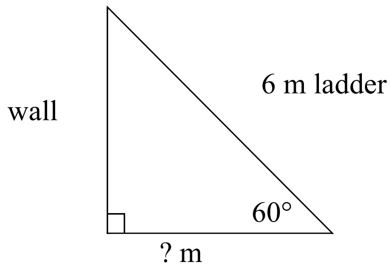
If you did not get these answers you should check that your calculator is working in degrees. It should have DEG written on the display.

Go back now and check your ratios for cosine and tangent in the last section. Find the cos and tan for the angle that you measured and check that your answers agree.

Example

A 6 m ladder leans against a wall such that it makes an angle of 60° with the ground. How far is the bottom of the ladder away from the wall?

Firstly, draw a diagram.



Next you need to choose the appropriate trig. ratio. We know the hypotenuse (the length of the ladder), the angle (60°) and wish to find the length of the side adjacent to the angle (the length along the ground to the foot of the ladder). So we must choose the cosine ratio.

$\cos 60° : 1 \equiv \text{adjacent} : \text{hypotenuse}$

$0.5 : 1 \equiv ? : 6$ On your calculator $\cos 60° = 0.5$

$0.5 \times 6 : 1 \times 6 \equiv 3 : 6$

Hence the foot of the ladder is 3 m from the wall.

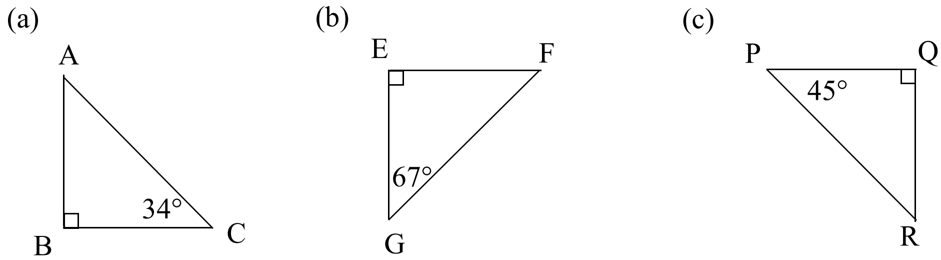
Activity 4.7

1. For each of the following right angled triangles (not drawn to scale) name the:

   (i) hypotenuse

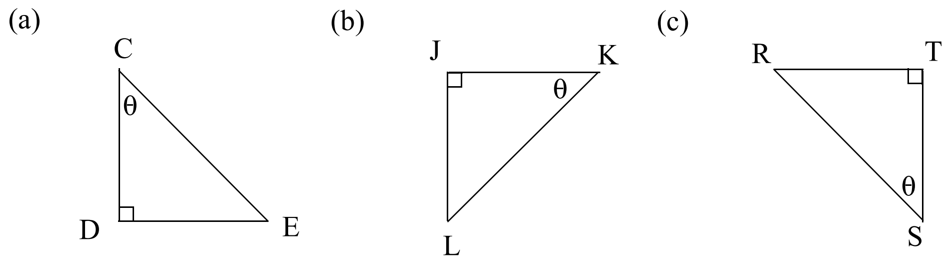
   (ii) side opposite the marked angle

   (iii) side adjacent to the marked angle.



2. Using your calculator find the ratios, sin, cos and tan, for each of the angles marked in the triangles in question 1. Round your answers to three decimal places if necessary.

3. For the following triangles **name the ratio** given for the marked angle.

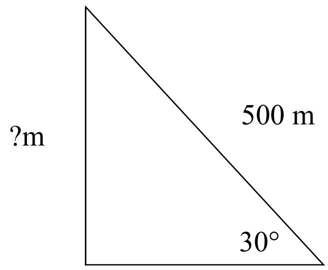


$\frac{CD}{CE} =$ $\frac{JL}{KL} =$ $\frac{RT}{ST} =$

$\frac{DE}{CE} =$ $\frac{JL}{JK} =$ $\frac{ST}{RS} =$

$\frac{DE}{CD} =$ $\frac{JK}{LK} =$ $\frac{RT}{RS} =$

4. A missile was fired at an angle of elevation of 30°. What is its height after it has travelled 500 m?



5. Shadows tell us that the sun's rays land on earth at an angle. This angle varies throughout the day. Find the height of a skyscraper, if the length of its shadow is 150 m when the angle of elevation of the sun is 45°. Draw a diagram to represent this situation.

4.2.3 Rates

So far we have only looked at comparing quantities of the same kind. A **rate** is a comparison of two quantities of **different kinds**. Unlike ratios, rates are expressed with the units included. Rates can be compared by expressing them in the same unit.

Example

A small car is driven 150 kilometres on 10 litres of petrol. What is the rate of petrol consumption.

We are comparing a **distance** of 150 kilometres with a **volume** of petrol of 10 litres.

Notice that in this case the units do not cancel

Rate of consumption $= \dfrac{150\text{ km}}{10\text{ L}} = \dfrac{15\text{ km}}{1\text{ L}}$

We don't usually see a rate written like this.

We could write it as 15 km/L and say 15 kilometres per litre.

Or, recall from module 3 that moving a power from the top to the bottom or the bottom to the top of a fraction means that we change the sign of the index.

Recall:
Module 3
Section 3.2.2

In this case we have $\dfrac{15\text{ km}^1}{1\text{ L}^1}$

We could write this as 15 kmL^{-1}. We would still say 15 kilometres per litre. With most documents now being typed or written with a computer this is a symbolism that you will regularly see.

However we write it, 15 kilometres per litre sounds like pretty good fuel economy.

A common way that car manufacturers talk about the worth of their respective cars is to talk about the fuel economy. They express this as how many litres of petrol the car would use to travel 100 kilometres.

Let's go back to our small car above.

This time we wish to compare volume to distance.

$$\text{Fuel economy} = \frac{10 \text{ L}}{150 \text{ km}} = 0.06666667 \text{ L/km}$$

We need to express this as litres per 100 kilometres instead on 1 kilometre as we have.

Multiplying both units by 100 gives us

$$\begin{aligned}\text{Fuel economy} &= 0.06666667 \times 100 \text{ L}/1 \times 100 \text{ km} \\ &\approx 6.67 \text{ L}/100 \text{ km}\end{aligned}$$

A pretty good selling point.

Example

Rates are also a convenient way of making price comparisons between similar items.

Suppose you were looking for the best buy in laundry detergent. At the supermarket you found two brands that looked promising: Kwik Klean at $3.95 for 1.5 kg and Super Sudser at $5.30 for 2 kg. How can you compare value when they come in different size containers?

By converting each to a rate per kilogram we can make a comparison on equal terms.

$$\text{Rate per kilogram for Kwik Klean} = \frac{\$3.95}{1.5 \text{ kg}} \approx \$2.63/\text{kg}$$

$$\text{Rate per kilogram for Super Sudser} = \frac{\$5.30}{2 \text{ kg}} = \$2.65/\text{kg}$$

We can now see that Kwik Klean at $3.95 for 1.5 kg is the better buy.

(Of course, price is not the only consideration when purchasing a product. You may buy the larger one to save on packaging or it may have more environmentally friendly ingredients.)

Activity 4.8

1. Complete the following:

   (a) 300 kilometres in 6 hours is a rate of ________ kilometres per hour.

   (b) $27 for 9 metres is a rate of ________ dollars per metre.

   (c) 42 hectares in 7 days is a rate of ________ hectares per day.

   (d) 120 runs for 4 wickets is a rate of ________ runs per wicket.

   (e) $320 for 40 hours work is a rate of ________ dollars per hour.

2. Two athletes were checking their pulse rates after an exercise session. Craig measured his pulse for 10 seconds and counted 20 beats, while Keely counted 31 beats over 15 seconds. Compare the two athletes pulse rates by converting each to beats per minute which is the usual unit in which to measure pulse rate.

3. Fertiliser is to be spread over a 20 hectare paddock at the rate of 250 kilograms per hectare. How many tonnes of fertiliser should the farmer order?

4. A 600 m^2 suburban block of land was advertised for sale at $42 000, while an equally well situated block of area 0.1 hectares was priced at $50 000. Which was the better value for money on a dollars/square metre basis?

5. On 3 000 hectares of Gunnadoo the owners increased the carrying capacity of sheep by improving some of the pastures. If the carrying capacity increased from 5 to $6\frac{1}{2}$ sheep per hectare, how many more sheep was the area able to run?

6. Sound is said to travel at an approximate speed of 340 metres per second. An aircraft travelling at this speed is said to be travelling at Mach 1.

   (a) What is this expressed in kilometres per hour?

   (b) If an aircraft is travelling at Mach 2, what is this speed in metres per second and in kilometres per hour?

4.3 A taste of things to come

1. Following is a bankcard statement that we looked at in module 2.

The following is a bankcard account statement for:

J Citizen
Grubby Ave
Toowoomba 4350

Account number		Annual percentage rate	Daily percentage rate	**Opening balance A$**
		Cash advances: 16.450%		
Credit limit	Credit available	Annual percentage rate	Daily percentage rate	**$1246.73**
4000		**Purchases 16.450%**		

Date	Reference	Transaction details			Amount A$	
08 JAN97		TARGET 5041	TOOWOOMBA	AU	57.60	DB
13 JAN97		SHELL SUPERBARGAIN	TOOWOOMBA	AU	53.00	DB
16 JAN97		K MART 1029	TOOWOOMBA	AU	78.00	CR
21 JAN97		FOSSEYS 212	ROCKHAMPTON	AU	45.85	DB
26 JAN97		BIG W 0260	TOOWOOMBA	AU	57.51	DB
28 JAN97		CALTEX	TOOWOOMBA	AU	40.00	DB
29 JAN97		K MART 1029	TOOWOOMBA	AU	16.60	DB
03 FEB97		PAYMENT – THANK YOU			100.00	CR
04 FEB97		WYALLA PLAZA DAY NIGHT	TOOWOOMBA	AU	41.10	DB
05 FEB97		CREDIT CHARGE – PURCHASES			19.51	
05 FEB97		CONTRACT STAMP DUTY			.83	

Opening balance	Total credits (CR)	Total debits (DB)	Credit and other charges	**Closing balance A$**
$1 246.73	**$178**	**$311.66**	**$20.34**	**$1 400.73**

Past due	Due date	Min.payment due	Payment record	Date paid	Amount paid
.00	**28 FEB97**				$

(a) You will notice at the top of the statement that the annual percentage rate has been stated. Calculate the daily percentage rate, rounded to the nearest ten thousandth, and include this in the appropriate column at the top of the statement.

(b) If the minimum payment due is calculated by finding 3% of the closing balance, rounded to the nearest dollar, what is the minimum payment due for this statement?

2. Now let's look at a task that teachers and others face when trying to work out final marks for students at the end of the semester.

For example, if you are currently studying the unit ***Focus on Study*** you will be involved in three areas of study: self development, communications and mathematics. Each of these parts contributes the same amount to the final mark.

(a) What fraction of the final mark comes from each of the three areas? Express this fraction as a percentage.

(b) Let's, as an example, consider assignment S2.

It is to be marked out of 50 as described on the marking sheet in module S1.

Its weighting for the 'S' component of the unit is 10% according to the unit spec.

Finally, its weighting for the overall unit is 3%.

The 10% weighting of this assignment in the 'S' component was determined by the unit team leader. The 'S' component contributes $\frac{1}{3}$ of the total unit ***Focus on Study***.

So, $\frac{1}{3} \times 10\,\% = 3\frac{1}{3}\%$ which has been rounded to 3%.

Consider a student who receives 25 out of 50 for assignment S2.

Since this assignment contributes 10% towards the 'S' component we must convert this to a mark out of 10.

To do this we calculate $\frac{25}{50} \times 10\,\% = 5\%$

That is, $\frac{25}{50} = \frac{5\%}{10\%}$

So a person scoring $\frac{25}{50}$ on assignment S2 has scored 5% towards the 'S' component total.

But, only $\frac{1}{3}$ of this contributes to the final assessment for *Focus on Study*.

That is, $\frac{1}{3} \times 5\% \approx 1.7\,\%$

So, in other words, if someone gets $\frac{25}{50}$ for assignment S2, this is worth 1.7% of the final assessment.

For the following marks, calculate the percentage contributing to the final assessment in ***Focus on Study***.

(i) $\frac{30}{50}$

(ii) $\frac{20}{50}$

Did you notice that these small differences in marks only had a minor effect on the percentage contributing to the final assessment?

3. Following is the recipe that makes 12 big chocolate patty cakes.

 3 tablespoons of butter

 $\frac{1}{2}$ cup sugar

 1 cup S R flour

 2 tablespoons of cocoa

 2 eggs

 $\frac{1}{4}$ cup milk

Suppose that you needed to supply the local school with 30 patty cakes (you did not want to have any left in the house). How much of each ingredient would you need to make the 30 cakes?

You should now be ready to attempt questions 6–11 of Assignment 2A (see your Introductory Book for details). If you have any questions, please refer them to your course tutor.

4.4 Post-test

1. The highest mountain in Queensland is Mount Bartle Frere, between Cairns and Innisfail, which rises to 1 657 metres. Australia's highest mountain is Mount Kosciuszko which is 2 228 metres high. Compare the heights of the two mountains by subtraction and by division.

2. (a) Convert 65% to a fraction in its simplest form.

   (b) Convert 0.0004 to a percentage.

3. A student receives 21 out of 27 for one assignment and 24 out of 31 for another. Which assignment received the better marks?

4. The following table shows the interest received on a term deposit.

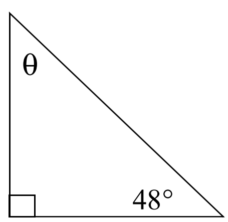
Term deposit interest rates					
Interest paid on maturity or annually if term exceeds 12 months					
	1 month	3 months	6 months	12 months	24 months
\$1 000	4.00% pa	4.50% pa	4.80% pa	5.00% pa	5.10% pa
\$5 000	4.50% pa	5.10% pa	5.50% pa	5.65% pa	5.85% pa
\$20 000	4.80% pa	5.20% pa	5.55% pa	5.70% pa	5.90% pa
\$50 000	5.25% pa	5.50% pa	5.60% pa	5.75% pa	5.95% pa

   How much interest will I receive for the following term deposits:

   (a) \$5 000 invested for 3 months?

   (b) \$20 000 invested for 2 years?

5.



   (a) How many degrees are there in a triangle?

   (b) Find the size of angle θ

   (c) Find $\sin\theta$, $\cos\theta$ and $\tan\theta$ for this triangle.

   Round your answers to 4 decimal places.

6. Following are the areas and populations of the states and Territories of Australia.

State or Territory	Area (km^2)	Population (1993)	Percentage of Australia	
			Area	Population
New South Wales (and ACT)	804 000	6 298 000		
Northern Territory	1 346 200	168 000		
Queensland	1 727 200	3 095 000		
South Australia	984 000	1 460 000		
Tasmania	67 800	472 000		
Victoria	227 600	4 461 000		
Western Australia	2 525 500	1 673 000		
Australia				

(a) Compare the areas of Queensland and Victoria by subtraction and by division.

(b) Compare the populations of Queensland and Victoria by subtraction and by division.

(c) Calculate the total area of Australia.

(d) Calculate the total population of Australia in 1993.

(e) Calculate the percentage of Australia that is represented by each state according to area and population (round your answers to the nearest tenth of a percent).

Check that your percentages add up to 100%. If they do not add to 100%, check your calculations and give an explanation for your result.

(f) Calculate the population rates for Queensland and Victoria in people per square kilometre.

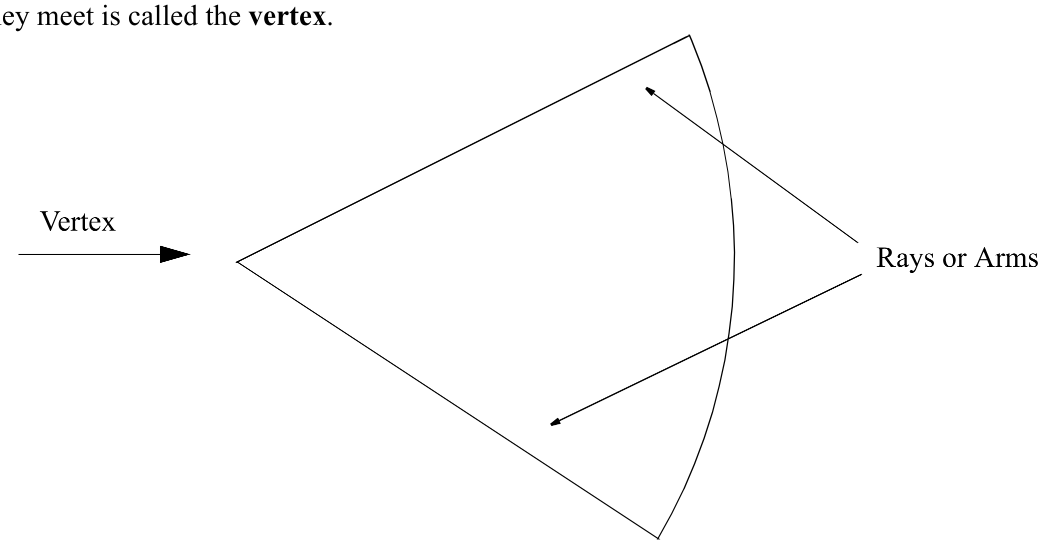
(g) Write a few sentences comparing the populations and areas of Queensland and Victoria.

4.5 Appendix: using a protractor

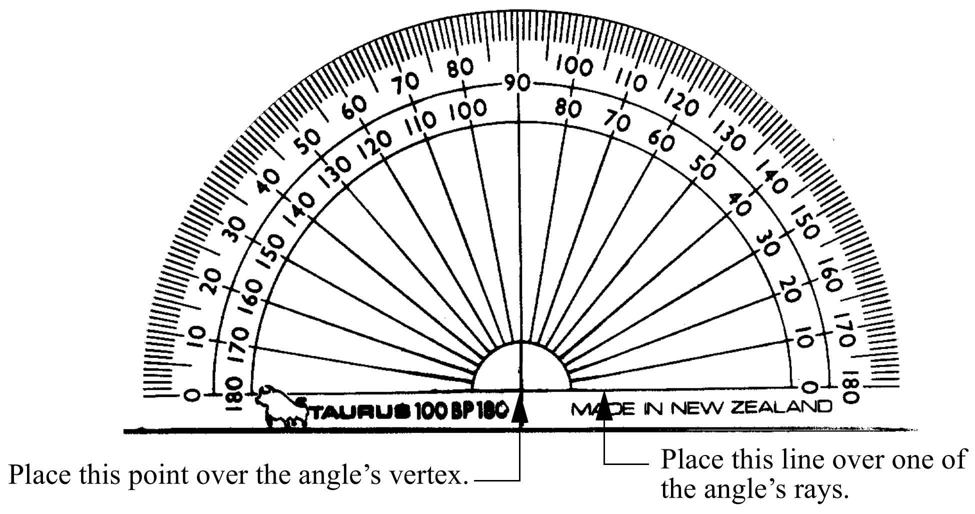
The Babylonians devised a method for measuring angles by dividing a circle into 360 equal parts called **degrees**. We can then divide the degree into smaller parts but that is beyond the scope of this module.

Measuring Angles

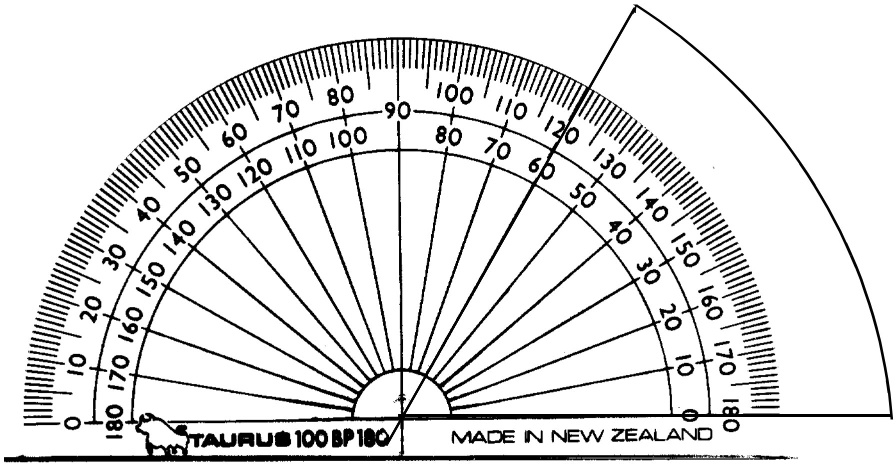
Imagine a pizza being cut into eight pieces. One piece would look like this. We call this shape a **sector** of the circle. The two straight sides of the slice form an angle and the point at which they meet is called the **vertex**.



A **protractor** is a device for measuring angles.



Let's measure the angle of the slice of pizza above. To measure the angle, place the centre of the protractor on the vertex of the angle and line up one of the arms of the angle with the baseline of the protractor. It will sometimes be necessary to extend the length of the arms of the angle so as to be able to read off the angle on the protractor.



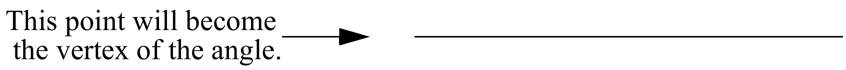
What did you get for the angle of the pizza sector: ....

If you said 60° then well done! If not, go back and have another look at your protractor. When measuring the angle you should always start from the zero on the protractor. Be sure that you are reading the angle from the correct scale.

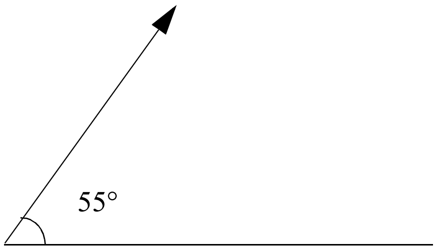
Constructing angles

Let's construct an angle of 55°

Firstly we must have a line on which to construct the angle.



Place the protractor over the line with its centre on the angle's vertex. Find the 55° mark on the protractor and make a mark on the page. Remove the protractor and join the mark to the vertex. You have constructed an angle of 55°



4.6 Solutions

Solutions to activities

Activity 4.1

1. We can see that the Blue Groper is the larger of the two fish.

That is, 150 cm – 50 cm = (150 – 50) cm = 100 cm

The Blue Groper is 100 cm longer than the Bream.

Or we could say that the Bream is 100 cm shorter than the Blue Groper.

Comparing the sizes of the fish be dividing gives us:

$$\frac{\text{Blue Groper Length}}{\text{Bream Length}} = \frac{150\cancel{cm}}{50\cancel{cm}} = 3$$

The cm's cancel.
The larger length is on the top.

Note that we have the Blue Groper length on the top of the fraction and so we can say that the Blue Groper is 3 times as long as the Bream.

If we put the Bream on top of the fraction we get:

$$\frac{\text{Bream Length}}{\text{Blue Groper Length}} = \frac{50\cancel{cm}}{150\cancel{cm}} = \frac{1}{3}$$

The cm's cancel.

We could then say that the Bream is $\frac{1}{3}$ of the size of the Blue Groper.

2. Blondes have about 1.4×10^5 hairs on their head while red heads have about 9×10^4

By subtraction: $1.4 \times 10^5 - 9 \times 10^4$

$= 1.4 \times 10^5 - 0.9 \times 10^5$

$= (1.4 - 0.9) \times 10^5$

$= 0.5 \times 10^5$

$= 5 \times 10^4$ Writing in scientific notation.

Blondes have 5×10^4 or 50 000 more hairs on their head than red heads.

By division:

$$(1.4 \times 10^5) \div (9 \times 10^4)$$

$$= \frac{1.4 \times 10^5}{9 \times 10^4}$$

$$\approx 0.156 \times 10^{5-4}$$

$$= 0.156 \times 10^1$$

$$= 1.56$$

Blondes have 1.56 times as many hairs on their head as do red heads.

3. White paint = 2 cans Red paint = 3 cans

By subtraction: 3 cans – 2 cans = (3 – 2) cans = 1 can

This pink paint requires 1 more can of red paint than white paint.

By division: $\frac{\text{Red Paint}}{\text{White Paint}} = \frac{3 \text{ cans}}{2 \text{ cans}} = 1.5$

This pink paint requires 1.5 times as much red paint as white paint.

4. Robert Wadlow 2.72 metres = 272 centimetres

Gul Mohammed 57 centimetres

Comparison by subtraction: 272 cm – 57 cm = (272 – 57) cm = 215 cm

Robert Wadlow is 215 cm or 2.15 m taller than Gul Mohammed.

Comparison by division: $\frac{272 \text{ cm}}{57 \text{ cm}} = 4.77$

Robert Wadlow is 4.77 times as tall as Gul Mohammed.

5. This question refers back to the original comparison by division.

$$\frac{\text{Speed of Light}}{\text{Speed of Sound}} = \frac{1.1 \times 10^9 \text{ kmh}^{-1}}{1.2 \times 10^3 \text{ kmh}^{-1}}$$

$$\approx 0.916\ 667 \times 10^{9-3}$$

$$= 0.916667 \times 10^6$$

$$= 9.166\ 67 \times 10^5$$

Light is travelling approximately $9.166\ 67 \times 10^5$ times faster than sound.

Activity 4.2

1.

(a) $8 \text{ out of } 10 = \frac{8}{10} \times 100\ \% = 80\%$

(b) $250 \text{ mL out of } 400 \text{ mL} = \frac{250}{400} \times 100\ \% = 62.5\%$

(c) $800 \text{ g out of } 2000 \text{ g} = \frac{800}{2000} \times 100\ \% = 40\%$

(d) $25 \text{ cm out of } 80 \text{ cm} = \frac{25}{80} \times 100\ \% = 31.25\%$

(e) $\$25 \text{ out of } \$60 = \frac{25}{60} \times 100\ \% \approx 41.7\%$

(f) $50 \text{ mL out of } 2 \text{ L} = \frac{50}{2000} \times 100\ \% = 2.5\%$

(g) 2×10^4 light years out of 3.5×10^3

$$\begin{aligned} &= \frac{2 \times 10^4}{3.5 \times 10^3} \times 100\% \\ &= (0.5714 \times 10^{4-3}) \times 100\% \\ &\approx 571.4\% \end{aligned}$$

2. The percentage passing $= \frac{45}{50} \times 100\% = 90\%$

   Therefore 90% of the students pass.

3. Percentage living to breed $= \frac{3}{1800} \times 100\% \approx 0.17\%$

   The number of turtles living long enough to breed is only 0.17% of the eggs laid.

   Sometimes you will see this written as 0.17 of one percent, emphasising that this percentage is less than one percent.

4.

Cereal	Number of people	Percentage of people
Corn Flakes	50	$\frac{50}{200} \times 100\% = 25\%$
Rice Bubbles	42	$\frac{42}{200} \times 100\% = 21\%$
Nutri Grain	39	$\frac{39}{200} \times 100\% = 19.5\%$
Rolled Oats	23	$\frac{23}{200} \times 100\% = 11.5\%$
Muesli	11	$\frac{11}{200} \times 100\% = 5.5\%$
Coco Pops	10	$\frac{10}{200} \times 100\% = 5\%$
Other Cereals	25	$\frac{25}{200} \times 100\% = 12.5\%$
	200	**100%**

5.

(a) $\frac{\text{Number Killed}}{\text{Total involved in accidents}} \times 100\% = \frac{55}{994} \times 100\% \approx 5.5\%$

Approximately 5.5% of pedestrians involved in accidents are killed.

(b) $\frac{\text{Taken to Hospital}}{\text{Total Alcohol Linked}} \times 100\% = \frac{65}{135} \times 100\% \approx 48.1\%$

Approximately 48.1% of people involved in alcohol related accidents are taken to hospital.

(c) The most common way that pedestrians are killed is by crossing the road where there is no pedestrian control.

$\frac{\text{Number Killed Crossing Road}}{\text{Total Number Killed}} \times 100\% = \frac{30}{55} \times 100\% \approx 54.5\%$

Approximately 54.5% of pedestrian deaths are caused by crossing the road where there is no pedestrian control.

(d) Of the 9 people killed walking near traffic, seven are killed walking with the traffic and only two are killed walking against the traffic as recommended. That is, about 78% are killed disobeying the childhood rules. This would suggest that this rule still holds true today.

Activity 4.3

1.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{2}{5}$	0.4	40%
$\frac{1}{4}$	0.25	25%
$\frac{4}{5}$	0.8	80%
$\frac{3}{8}$	0.375	37.5%
$\frac{78}{100} = \frac{39}{50}$	0.78	78%
$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$

2.

(a) 12% of 250 mL $= \frac{12}{100} \times 250$ mL $= 30$ mL

(b) 90% of $\$75 = \frac{90}{100} \times \$75 = \$67.50$

(c) 30% of 645 g $= \frac{30}{100} \times 645$ g $= 193.5$ g

(d) 250% of $\$16.40 = \frac{250}{100} \times \$16.40 = \$41.00$

(e) 2% of 900 mL $= \frac{2}{100} \times 900$ mL $= 18$ mL

(f) 15.6% of 300 mL $= \frac{15.6}{100} \times 300$ mL $= 46.8$ mL

(g) 5.5% of 350 g $= \frac{5.5}{100} \times 350$g $= 19.25$ g

(h) 70.5% of 400 mL $= \frac{70.5}{100} \times 400$mL $= 282$ mL

3. Injuries preventable $= \frac{60}{100} \times 6100 = 3660$

   Therefore, 3 660 injuries could be prevented with the use of child-restraint seats.

4. For a person 165 cm tall the length of the femur will equal:

   $$\frac{27.5}{100} \times 165 \text{ cm} = 45.375 \text{ cm}$$

5. Percentage increase in cattle $= \frac{\text{Number of increase}}{\text{Original number}} \times 100\%$

   $= \frac{200}{100} \times 100\%$

   $= 200\%$

   Percentage increase in sheep $= \frac{\text{Number of increase}}{\text{Original number}} \times 100\%$

   $= \frac{400}{500} \times 100\%$

   $= 80\%$

   In 1997 the cattle numbers were increased by 200% while the sheep numbers were increased by only 80%.

6.

Date	Particulars	Debit	Credit	Balance
1 July				62 347.65
20 July	Loan payment		865.00	61 482.65
31 July	Interest	432.08		61 914.73
20 August	Loan payment		865.00	61 049.73
31 August	Interest	429.06		61 478.79
20 September	Loan payment		865.00	60 613.79
30 September	Interest	412.41		61 026.20

To calculate interest on 31 August:

From 1st August to 20 August (20 days) the balance is \$61 914.73

From 21st August to 31 August (11 days) the balance is \$61 049.73

Let's consider the first balance. The interest for the whole year on this balance would be:

$\$61\,914.73 \times \frac{8.2}{100} = \$5\,077.00786$ Never round off until the end.

Now work out the daily rate: $\$5\,077.00786 \div 365 = \$13.90961058.....$

Calculate for the number of days owing. $\$13.90961058 \times 20 = \$278.1922115.....$

Now do the same for the other amount.

Yearly: $\$61049.73 \times \frac{8.2}{100} = \$5006.07786$

Daily: $\$5006.07786 \div 365 = \$13.71528181$

For 11 days: $\$13.71528181 \times 11 = \$150.8680999$

Now add the two amounts together to find the total interest charged.

$\$278.1922115 + \$150.8680999 = \$429.0603114....$

Now we can round off these numbers. The amount of interest charged is $429.06

To calculate interest on 30 September:

From 1st September to 20 September (20 days) the balance is $61 478.79

From 21st September to 30 September (10 days) the balance is $60 613.79

Let's consider the first balance. The interest for the whole year on this balance would be:

$\$61478.79 \times \frac{8.2}{100} = \$5041.26078$ Never round off until the end.

Now work out the daily rate: $\$5041.26078 \div 365 = \$13.81167337$

Calculate for the number of days owing. $\$13.81167337 \times 20 = \$276.2334674$.

Now do the same for the other amount.

Yearly: $\$60613.79 \times \frac{8.2}{100} = \$4970.33078$

Daily: $\$4970.33078 \div 365 = \$13.6173446$

For 10 days: $\$13.6173446 \times 10 = \$136.173446$

Now add the two amounts together to find the total interest charged.

$\$276.2334674 + \$136.173446 = \$412.4069134....$

Now we can round off these numbers. The amount of interest charged is $412.41

7.

(a) Sales workers are the group projected to show the greatest increase in employment numbers.

(b) The numbers employed in this area have decreased. When talking about increases in numbers we refer to a decrease as a **negative increase**.

(c) Difference between highest and lowest growth areas

$= 48.5 - {}^{-}1.9$

$= 48.5 + 1.9$

$= 50.4$

(d) Percentage increase in number of managers

$$= \frac{\text{Amount of increase}}{\text{Original Amount}} \times 100\%$$

$$= \frac{1238\ 100 - 884\ 200}{884\ 200} \times 100\%$$

$$= \frac{353\ 900}{884\ 200} \times 100\%$$

$$\approx 40.0\%$$

Therefore managers are expected to increase by about 40%

(e) Percentage increase in numbers of stationary plant operators

$$= \frac{\text{Amount of increase}}{\text{Original Amount}} \times 100\%$$

$$= \frac{49\ 000 - 52\ 700}{52\ 700} \times 100\%$$

$$= \frac{-3\ 700}{52\ 700} \times 100\%$$

$$\approx -7.0\%$$

It appears that the calculation in the table is incorrect.

(f) Percentage increase in total numbers of machine operators:

$$= \frac{\text{Amount of increase}}{\text{Original Amount}} \times 100\%$$

$$= \frac{541\ 600 - 552\ 400}{552\ 400} \times 100\%$$

$$= \frac{-10\ 800}{552\ 400} \times 100\%$$

$$\approx -2.0\%$$

It appears that the calculation in the table is incorrect. This time the error appears to be because of incorrect rounding.

8.

(a) There are 300 birds in the aviary.

Number of parrots: $\frac{25}{100} \times 300 = 75$

Number of finches: $\frac{45}{100} \times 300 = 135$

Number of pigeons: $\frac{30}{100} \times 300 = 90$

Check that you have accounted for all the birds. $75 + 135 + 90 = 300$

Activity 4.4

1.

(a) Angle for rare: $\frac{5}{100} \times 360° = 18°$

Angle for medium/rare: $\frac{19}{100} \times 360° \approx 68°$

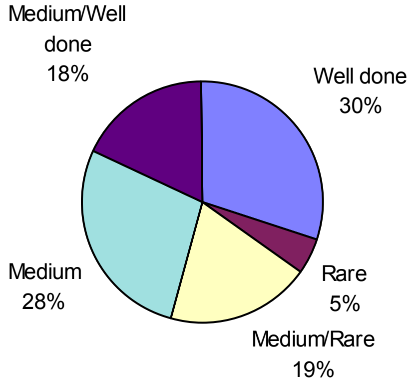
Angle for medium: $\frac{28}{100} \times 360° \approx 101°$

Angle for medium/well done: $\frac{18}{100} \times 360° \approx 65°$

Angle for well done: $\frac{30}{100} \times 360° \approx 108°$

Check to see that we have 360°. $18° + 68° + 101° + 65° + 108° = 360°$

Degree of Doneness



(b) Percentage ordering medium $= \frac{28}{100} \times 3\ 450 = 966$

Therefore, 966 patrons would order their steak medium.

(c) Percentage unhappy $= \frac{31}{100} \times 3\ 450 = 1\ 069.5$

We cannot have a part of a person, so we say that about 1 070 people were unhappy with their steak.

2.

(a) Percentage opposed to change: $\frac{126}{360} \times 100\% = 35\%$

Percentage in favour of change: $\frac{162}{360} \times 100\% = 45\%$

Percentage undecided: $\frac{72}{360} \times 100\% = 20\%$

Check that your percentages add to 100%. Yes, they do!

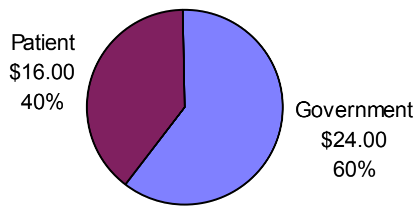
(b) 35% of teachers opposed the change.

That is, $\frac{35}{100} \times 40 = 14$

Therefore 14 of the 40 teachers surveyed opposed the change of hours.

3.

(a) While the data itself on this graph is accurate, the impression given does not match the data. If the data were rearranged into a two dimensional pie chart it would look like this:



So to comment on the pie chart, the sectors do not look to be the correct size and the centre of the pie chart is not in the correct position.

(b) The information presented in this graph is not suitable for a pie chart. A pie chart is used to represent 100% of something. The percentages on this graph do not add to 100%. Also, all sectors appear to be the same size, but each is representing a different percentage.

Activity 4.5

1.

(a) 21 : 15 ≡ 7 : 5

(b) 20 : 12 ≡ 5 : 3

(c) 350 g to 500 g: 350 : 500 ≡ 7 : 10

(d) 90 mL to 300 mL: 90 : 300 ≡ 3 : 10

(e) 5 mL to 3 L = 5 mL to 3 000 mL: 5 : 3 000 ≡ 1 : 600

(f) 5 g to 10 kg = 5 g to 10 000 g: 5 : 10 000 ≡ 1 : 2 000

(g) 2.2 kg to 500 g = 2 200 g to 500 g: 2 200 : 500 ≡ 22 : 5

(h) 9.5 L to 50 mL = 9 500 mL to 50 mL: 9 500 : 50 ≡ 190 : 1

2.

(a) (i) 8 incisors

(ii) 4 canines

(iii) 8 bicuspids

(iv) 12 molars

Check that you have accounted for all 32 teeth.

(b) (i) incisors to molars 8 : 12 ≡ 2 : 3

(ii) bicuspids to canines 8 : 4 ≡ 2 : 1

(iii) incisors to bicuspids 8 : 8 ≡ 1 : 1

(iv) molars to all teeth 12 : 32 ≡ 3 : 8

(v) incisors to molars to all teeth. 8 : 12 : 32 ≡ 2 : 3 : 8

3.

(a) Fraction of the games won = $\frac{24}{30} = \frac{4}{5}$

(b) 6 games are lost.

(c) Ratio of games lost to games won: 6 : 24 ≡ 1 : 4

4. If 35% of people skip breakfast then 65% must eat breakfast.

Ratio of people who eat breakfast to those who skip breakfast: 65 : 35 ≡ 13 : 7

5.

(a) $8 : 5 \equiv 8 \times 3 : 5 \times 3 \equiv 24 : 15$

(b) $3 : 4 \equiv 3 \times 5 : 4 \times 5 \equiv 15 : 20$

(c) $4 : 6 \equiv 4 \times \frac{10}{4} : 6 \times \frac{10}{4} \equiv 10 : 15$

(d) $7 : 3 \equiv 7 \times \frac{4}{3} : 3 \times \frac{4}{3} \equiv 9\frac{1}{3} : 4$

(e) $8 : 4 \equiv 2 : 1$

(f) $\frac{4}{7} : \frac{3}{5} \equiv \frac{4}{7} \times \frac{2}{3} : \frac{3}{5} \times \frac{2}{3} \equiv \frac{8}{21} : \frac{6}{15}$

(g) $2 : 0.75 \equiv 2 \div 3 : 0.75 \div 3 \equiv 0.\dot{6} : 0.25$

6. Loam to peat to sand

$7 : 4 : 2$

To get to 4 buckets of sand we have multiplied by 2.

$7 \times 2 : 4 \times 2 : 2 \times 2$

$\equiv 14 : 8 : 4$

Therefore if we use 4 buckets of sand we must use 14 buckets of loam and 8 buckets of peat.

7. The measured distance between B and C on the map was 4.3 cm.

$1 : 1\,000\,000 \equiv 4.3 : ?$

What did you multiply 1 by to get 4.3? You multiplied by 4.3.

$1 \times 4.3 : 1\,000\,000 \times 4.3 \equiv 4.3 : 4\,300\,000$

So the actual distance between B and C is 4 300 000 cm.

We should convert it to kilometres, a more appropriate measure.

$4\,300\,000 \text{ cm} = 43\,000 \text{ m} = 43\,000 \times 10^{-3} \text{ km} = 43 \text{ km}$

The distance between towns B and C is 43 kilometres.

8. Remember a scale like this represents the scale drawing to the original. Since centimetres would be a good measure in which to draw our scale diagram let's convert the lengths to centimetres.

   14 m = 1 400 cm

   26 m = 2 600 cm

   Now,
   $$1 : 500 \equiv ? : 1\,400$$
   $$1 : 500 \equiv 1 \times 2.8 : 500 \times 2.8$$
   $$\equiv 2.8 : 1\,400$$

   and,
   $$1 : 500 \equiv ? : 2\,600$$
   $$1 : 500 \equiv 1 \times 5.2 : 500 \times 5.2$$
   $$\equiv 5.2 : 2\,600$$

   The scale drawing of the basketball court will be 2.8 cm by 5.2 cm.

9. The ratio patients to nurses is 12 : 2

   If there are 70 patients, we are finding $12 : 2 \equiv 70 : ?$
   $$12 : 2 \equiv 12 \times \frac{70}{12} : 2 \times \frac{70}{12}$$
   $$\equiv 70 : 11.\dot{6}$$

   If there were 70 patients the hospital would need 12 nurses.

Activity 4.6

1. Males to Females are in the ratio $4 : 3 \equiv 32 : ?$

$$4 \times 8 : 3 \times 8 \equiv 32 : 24$$

Therefore there are 24 females in the class.

2. In the ratio 13 : 5 there are 18 parts. So $\frac{13}{18}$ of the stock are sheep and $\frac{5}{18}$ are cattle.

Number of cattle $= \frac{5}{18} \times 1\,980 = 550$

In 1970 the number of cattle on Gunnadoo was 550.

3. In the ratio 2 : 3 : 4 there are 9 parts. So $\frac{2}{9}$ of the bag of fertilizer is nitrogen, $\frac{3}{9}$ is potash and $\frac{4}{9}$ phosphate.

Amount of nitrogen $= \frac{2}{9} \times 27 = 6$

There will be 6 kilograms of nitrogen in the 27 kilogram bag.

4. Since there are 5 parts in the ratio, Daniel will receive $\frac{3}{5}$ and Tom $\frac{2}{5}$.

Daniel receives: $\frac{3}{5} \times \$10 = \$6$

Tom receives: $\frac{2}{5} \times \$10 = \$4$

5. The sample has a ratio of red to white cells of 4 250 000 : 2 125

If we were to make this into a ratio to 1 we would divide both sides by 2 125.

$$4\,250\,000 \div 2\,125 \quad : \quad 2\,125 \div 2\,125$$

$$2000 \quad : \quad 1$$

This sample of blood has a much higher red cell count than is normal. In fact there are twice as many red cells to white cells than for normal blood.

6.

(a)	An airline pilot:	1 in 1 100	1 : 1 099
(b)	In mining and agriculture:	1 in 2 300	1 : 2 299
(c)	In transportation:	1 in 4 500	1 : 4 499
(d)	In government:	1 in 11 000	1 : 10 999

(e)	In manufacturing	1 in 23 000	1 : 22 999
(f)	In sales	1 in 24 000	1 : 23 999
(g)	In office work	1 in 37 000	1 : 36 999

7.

(a) Operations to deaths are in the ratio 3 800 : 5. We could simplify this to be 760 : 1

If 10 500 operations are performed, we are looking to evaluate:

$$760 : 1 \equiv 10\ 500 : ?$$
$$760 \times \frac{10\ 500}{760} : 1 \times \frac{10\ 500}{760}$$
$$10\ 500 : 13.816$$

In this particular year we would expect the 10 500 operations to result in about 14 deaths.

(b) We are trying to evaluate: $760 : 1 \equiv ? : 75$

$$760 \times 75 : 1 \times 75 \equiv 57\ 000 : 75$$

If there had been 75 deaths it would be expected that there had been 57 000 operations.

Activity 4.7

1. (a) (i) hypotenuse AC
   (ii) side opposite the marked angle AB
   (iii) side adjacent to the marked angle. BC
   (b) (i) hypotenuse FG
   (ii) side opposite the marked angle EF
   (iii) side adjacent to the marked angle. EG
   (c) (i) hypotenuse PR
   (ii) side opposite the marked angle QR
   (iii) side adjacent to the marked angle. PQ

   Note that you could have written the letters in these answers around the other way.

2.

(a)	(b)	(c)
$\sin 34° \approx 0.559$	$\sin 67° \approx 0.921$	$\sin 45° \approx 0.707$
$\cos 34° \approx 0.829$	$\cos 67° \approx 0.391$	$\cos 45° \approx 0.707$
$\tan 34° \approx 0.675$	$\tan 67° \approx 2.356$	$\tan 45° = 1$

3.

(a)

$\frac{CD}{CE}$ = cosine

$\frac{DE}{CE}$ = sine

$\frac{DE}{CD}$ = tangent

(b)

$\frac{JL}{KL}$ = sine

$\frac{JL}{JK}$ = tangent

$\frac{JK}{LK}$ = cosine

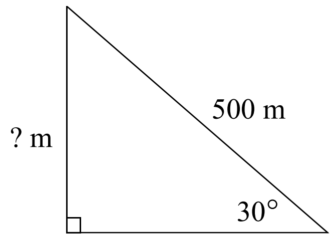
(c)

$\frac{RT}{ST}$ = tangent

$\frac{ST}{RS}$ = cosine

$\frac{RT}{RS}$ = sine

4. We will need to use the sine ratio.



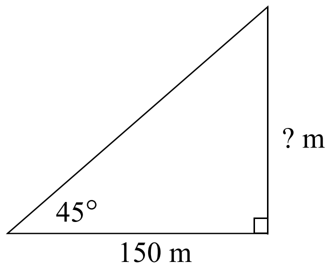
sin 30° : 1 ≡ opposite : hypotenuse

0.5 : 1 ≡ ? : 500

0.5 × 500 : 1 × 500 ≡ 250 : 500

The missile is 250 metres high.

5. We will need to use the tangent ratio.



tan 45° : 1 ≡ opposite : adjacent

1 : 1 ≡ ? : 150

1 × 150 : 1 × 150 ≡ 150 : 150

So the height of the building must have been 150 m.

Activity 4.8

1.

(a) 300 kilometres in 6 hours is a rate of 50 kilometres per hour.

(b) $27 for 9 metres is a rate of 3 dollars per metre.

(c) 42 hectares in 7 days is a rate of 6 hectares per day.

(d) 120 runs for 4 wickets is a rate of 30 runs per wicket.

(e) $320 for 40 hours work is a rate of 8 dollars per hour.

2. Craig's pulse rate is 20 beats for 10 seconds

= 20 × 6 beats for 60 seconds

= 120 beats per minute.

Keely's pulse rate is 31 beats for 15 seconds

$= 31 \times 4$ beats for 60 seconds

$= 124$ beats per minute.

After the exercise session, Keely's pulse rate is slightly higher than Craig's.

3. If the farmer needs 250 kg per hectare for 20 hectares, 250×20 kg are needed.

   The farmer should order 5 000 kg or 5 tonnes of fertiliser.

4. The first block is \$42 000 for 600 m^2

   which gives \$70/square metre.

   The second block is \$50 000 for 0.1 hectares

   which is \$50 000 for 1 000 m^2

   which gives \$50/square metre.

   The second block of land appears to be the better buy.

5. On 3 000 hectares at 5 sheep/hectare, there would be 15 000 sheep.

   On 3 000 hectares at 6.5 sheep/hectare, there would be 19 500 sheep.

   The increase in sheep numbers = 19 500 – 15 000 = 4 500

6.

(a)	340 metres	in	1 second	convert to minutes
	340×60 metres	in	1×60 seconds	multiply both sides by 60
	20 400 metres	in	60 seconds (1 minute)	convert to hours
	$20\,400 \times 60$ metres	in	1×60 minutes	multiply both sides by 60
	1 224 000 metres	in	60 minutes (1 hour)	
	1 224 kilometres	in	1 hour	convert metres to kilometres

Therefore, 340 m/s is equivalent to 1 224 km/h

(b) At Mach 2 the aircraft is travelling at twice the speed of sound.

That is, 680 m/s or 2 448 km/h

Solutions to a taste of things to come

1.

(a) If the annual percentage rate is 16.450 % then the daily rate will be:

$\frac{16.450\%}{365} \approx 0.0451\%$ rounded to the nearest ten thousandth.

(b) Closing balance is $1 400.73 so the minimum payment will be:

$\frac{3}{100} \times \$1\ 400.73 \approx \$42$ rounded to the nearest dollar

2.

(a) Since there are three areas contributing to the final mark, each must contribute 1/3 of the marks.

As a percentage $\frac{1}{3} = 33\frac{1}{3}\%$

(b) (i) A person getting $\frac{30}{50}$ on the assignment, would have:

$\frac{30}{50} \times 10\% = 6\%$ contributing to the 'S' Component and

$\frac{1}{3} \times 6\% = 2\%$ contributing to the final mark.

(ii) A person getting $\frac{20}{50}$ on the assignment, would have:

$\frac{20}{50} \times 10\% = 4\%$ contributing to the 'S' Component and

$\frac{1}{3} \times 4\% \approx 1.3\%$ contributing to the final mark.

3. We are making 30 cakes from a recipe for 12 and we could think about the answer before we begin. Thirty cakes is more than twice the recipe but not three times. So each of our answers should be 2 and a bit times, but not three times, the original amount for each ingredient.

Now:

number of cakes now : number of cakes needed $\equiv$ ingredients now : ingredients needed

Let's look at butter first:

$12 : 30 \equiv 3 : ?$ We have divided by 4

$12 \div 4 : 30 \div 4 \equiv 3 : 7.5$

Therefore we would need 7.5 tablespoons of butter.

Sugar: $12 : 30 \equiv \frac{1}{2} : ?$ We have divided by 24

$12 \div 24 : 30 \div 24 \equiv \frac{1}{2} : 1\frac{1}{4}$

Therefore we would need $1\frac{1}{4}$ cups of sugar.

Flour: $12 : 30 \equiv 1 : ?$ We have divided by 12

$12 \div 12 : 30 \div 12 \equiv 1 : 2\frac{1}{2}$

Therefore we would need $2\frac{1}{2}$ cups of flour.

Cocoa: $12 : 30 \equiv 2 : ?$ We have divided by 6

$12 \div 6 : 30 \div 6 \equiv 1 : 5$

Therefore we would need 5 tablespoons of cocoa.

Eggs: $12 : 30 \equiv 2 : ?$ We have divided by 6

$12 \div 6 : 30 \div 6 \equiv 1 : 5$

Therefore we would need 5 eggs.

Milk: $12 : 30 \equiv \frac{1}{4} : ?$ We have divided by 48

$12 \div 48 : 30 \div 48 \equiv \frac{1}{4} : \frac{5}{8}$

Therefore we would need $\frac{5}{8}$ of a cup of milk.

Solutions to post-test

1. By subtraction:

   2 228 m – 1 657 m = 571 m

   Therefore, Mount Kosciuszko is 571 m higher than Mount Bartle Frere.

   By division:

   $$\frac{2\ 228\ \text{m}}{1\ 657\ \text{m}} \approx 1.34$$

   Therefore, Mount Kosciuszko is about 1.34 times as high as Mount Bartle Frere.

2.

(a) $65\% = \frac{65}{100} = \frac{13}{20}$

(b) $0.0004 = 0.0004 \times 100\% = 0.04\%$

3. $\frac{21}{27} = \frac{21}{27} \times 100\% = 77\frac{7}{9}\% \approx 77.78\%$

   $\frac{24}{31} = \frac{24}{31} \times 100\% = 77\frac{13}{21}\% \approx 77.42\%$

   The marks are actually very close. The first assignment has a slightly better result.

4.

(a) Investing \$5 000 for three months gives an interest rate of 5.1% pa

$$\begin{aligned} \text{Interest for whole year would be } &= 5.1\% \times \$5000 \\ &= \frac{5.1}{100} \times \$5000 \\ &= \$255 \end{aligned}$$

Since the money is invested for 3 months, or $\frac{1}{4}$ of a year the interest will be $\frac{1}{4}$ of this amount.

Interest for 3 months $= \frac{1}{4} \times \$255 = \$63.75$

(b) Investing \$20 000 for 2 years gives an interest rate of 5.90% pa

Interest for one year would be $= 5.9\% \times \$20\,000$

$= \frac{5.9}{100} \times \$20\,000$

$= \$1\,180$

Since the money is invested for 2 years the total amount of interest will be:

$$\$1\,180 \times 2 = \$2\,360$$

5.

(a) There are 180° in a triangle.

(b) $\theta = 42°$ (since $90° + 48° + 42° = 180°$)

(c) $\sin 42° \approx 0.6691$ $\cos 42° \approx 0.7431$ $\tan 42° \approx 0.9004$

6.

State or Territory	**Area (km^2)**	**Population (1993)**	**Percentage of Australia**	
			Area	Population
New South Wales (and ACT)	804 000	6 298 000	10.5	35.7
Northern Territory	1 346 200	168 000	17.5	1.0
Queensland	1 727 200	3 095 000	22.5	17.6
South Australia	984 000	1 460 000	12.8	8.3
Tasmania	67 800	472 000	0.9	2.7
Victoria	227 600	4 461 000	3.0	25.3
Western Australia	2 525 500	1 673 000	32.9	9.5
Australia	**7 682 300**	**17 627 000**	**100.1**	**100.1**

(a) Area of Queensland = 1 727 200 km^2

Area of Victoria = 227 600 km^2

By subtraction: 1 727 200 km^2 – 227 600 km^2 = 1 499 600 km^2

Queensland is 1 499 600 km^2 greater in area than Victoria.

By division: $\frac{1\,727\,200 \text{ km}^2}{227\,600 \text{ km}^2} \approx 7.59$

Queensland is approximately 7.59 times larger in area than Victoria.

(b) Population of Queensland = 3 095 000

Population of Victoria = 4 461 000

By subtraction: 4 461 000 – 3 095 000 = 1 366 000

Victoria has 1 366 000 more people than does Queensland.

By division: $\frac{4\,461\,000}{3\,095\,000} \approx 1.44$

Victoria has approximately 1.44 times the population of Queensland.

If we looked at this the other way: $\frac{3\,095\,000}{4\,461\,000} \approx 0.69$

Queensland has approximately 0.69 times the population of Victoria.

(c) Total area of Australia = 7 682 300 km^2 (See table above)

(d) Total population in 1993 = 17 627 000 people. (See table above)

(e) See table above. Due to rounding off, the totals are 100.1% instead of 100% as they should be. To correct this you could change one of the answers that you have rounded up.

(f) Area of Queensland = 1 727 200 km^2 Population of Queensland = 3 095 000

Area of Victoria = 227 600 km^2 Population of Victoria = 4 461 000

For Queensland :

$$= \frac{3\,095\,000 \text{ people}}{1\,727\,200 \text{ km}^2}$$

$$= \frac{3\,095\,000}{1\,727\,200} \text{ people/km}^2$$

$$\approx 1.79 \text{ people/km}^2$$

For Victoria:

$$= \frac{4\,461\,000 \text{ people}}{227\,600 \text{ km}^2}$$

$$= \frac{4\,461\,000}{227\,600} \text{ people/km}^2$$

$$\approx 19.6 \text{ people/km}^2$$

For every square kilometre in Queensland we only have about 2 people, while in Victoria there are about 20 people for every square kilometre. (No wonder they want to move to Queensland.) You might like to think of this in comparison to India which has a population density of about 273 people per square kilometre.

(g) Queensland has a much greater area than Victoria, in fact the area of Queensland is about 7.59 times the area of Victoria. Queensland though, has 1 366 000 fewer people than Victoria. This leads to the population density of Victoria being about 20 people per square kilometre while Queensland has only about 2 people per square kilometre.